PITMAN'S HANDWORK SERIES

Handwork

and

Practical Arithmetic

BOOK I

By

George F. Johnson

Editor of "Educational Handwork," and Inspector of Handwork, Liverpool Education Committee

WITH AN INTRODUCTION BY

J. A. GREEN, M.A.

Professor of Education in the University of Sheffield

London

Sir Isaac Pitman & Sons, Ltd., 1 Amen Corner, E.C.

Bath and New York

1912

HANDWORK & ARITHMETIC.

Book 2. 106 pp., Price 2s. net.

PRINTED BY SIR ISAAC PITMAN & SONS, LTD., LONDON, BATH, AND NEW YORK

PREFACE

ONE of the most important principles underlying the newer education which is gradually taking root, is that the child himself shall be the chief agent in his own development. With this object in view the following pages have been written.

The method adopted throughout the book has been to provide suitable examples and conditions for the children, and by careful investigation to draw conclusions and formulate rules. The mass of information obtained by this method may not be so great as by the more formal method of "Instruction," but it has the sounder merit of stability. What one *does for oneself*, or *re-discovers for oneself*, makes a firmer and more lasting psychical impression than what one is only told or what one has only seen.

There are seven parts in the whole scheme, and these are arranged to correspond to the usual standards or classes of an elementary school. These parts may be modified to suit individual necessities, or the whole may be worked just as it stands.

Exhaustive the author does not claim that it is; but if it is taken up in the suggestive spirit in which it is written, and not hurried through in order to see "some results," then the pleasure and gratification of those using the book will be surprising. Experience has shown over and over again that the scoffers and doubters of a year or two ago are now the most ardent in favour of wider application of the principle of handwork in education.

The materials are inexpensive and do not require especial preparation: they are chiefly paper and cardboard. The exclusion of other media does not necessarily imply that paper and cardboard are *always* the best. Clay, sticks, etc., have a place, and when these media are thought to be most suitable for a certain purpose, no hesitation should be made in using them. In fact, *all* media have a place and purpose in Educational Handwork, the place being always quite subservient to the purpose. Too much emphasis cannot be given to this aspect of the work, for in its truest sense handwork is a *method* by which children obtain for themselves, through their own practical experiences, a knowledge of the great

facts of life. It must not be regarded as a separate additional *subject* on the proverbially overloaded time-table, but used as a means to an end in aiding the teaching of arithmetic.

The scheme outlined in the book has been adopted in the author's own town for three or four years, and has received high encomiums from inspectors and others in a position to judge of its merits. The author here begs to offer his best thanks to many kind friends for valuable suggestions and advice, and especially to Mr. C. H. Ince for his assistance in the preparation of the drawings for the illustrations.

GEO. F. JOHNSON.

INTRODUCTION

In accepting Mr. Johnson's invitation to write a short introductory note to his book on *Handwork and Arithmetic*, I had in mind certain principles which seem to me to lie at the root of sound work in this sphere.

It is, perhaps, worth while pointing out that Handwork preceded Arithmetic in the development of the race, that the very form of the hand has been the prime factor in determining the special convention under which men have chosen to arrange their calculating machinery. Had we been six-fingered animals, we may confidently suppose that we should have had what would have been in many ways a more convenient duodecimal system of notation. However that may be, the fact remains that it was with the development of manual power that the idea of number developed. We are all familiar with the story of the savage races who barter their goods to the covetous white man for sticks of tobacco, which they count so carefully by laying them upon their fingers that they pay no attention to the size of the sticks. We do not need to go so far afield to see something of the same sort. A child of three will commonly take three pieces of chocolate rather than two, though the difference in number is more than made up by the difference in size. Herein lies the problem. We may teach a child to count, to add and subtract, to multiply and divide, without a single reference to the situations which made arithmetic first of all a necessity, and afterwards an instrument of precision in matters of quantity. We still use it in both ways, but there is all the difference in the world between three sticks of tobacco or three pieces of chocolate, without any reference to standard size, and three sticks of accepted dimensions.

Messrs. McLelland and Dewey suggest that number grew out of measurement, and in a sense it is true, but I have no doubt myself that rough quantitative terms and number signs were used long before the idea of precise measurement arose and came finally to be fundamental—making arithmetic possible. So there seems no reason why we should not teach a child to count before we teach him to measure, though it is only when we teach him to bring his

symbols to bear upon the preciser estimation of quantities that we are laying the foundations of mathematics. The situation in which this necessity arises most naturally and most easily is in work with the hand. Here rough approximations soon cease to be sufficient; we must be exact if we would be successful; we must measure: Standards, units must be used; and when we say one or two or seven we must know whether it is metres or yards or shillings to which we refer.

The progress which has been made in recent years in thus relating Arithmetic to Handwork is a most welcome sign, though we may still do much more to relieve the schoolboy of the type of arithmetic which led him to Stocks and Shares, International Exchange, and other high financial matters that concern a favoured few, but left him ignorant of the use of arithmetic in the constructive exercises which come the way of every man. Under the newer methods of approach, fractions are no longer inventions for racking the brains of indifferent schoolboys. Long measure and square measure are not confined in their interest to the task of getting right answers to the sums in exercise this or that, and an early practical acquaintance with the metric system is so convincing of its superiority, that we may hope thereby to expedite its adoption in the commerce of the country.

"Whatsoever thy hand findeth to do, do it with thy might." This attitude of practical purposefulness is best realised in early life—and, indeed, in the lives of most grown men—in the employment of the hands; as he in this connection sees the increase in his total effectiveness which command over arithmetic gives, the schoolboy's outlook changes. His books take on a new meaning, and his figures are endowed with life.

It is as contributing to the realisation of one's ideals in this subject that I welcome with joy this interesting and thoroughly practical book.

J. A. G.

THE UNIVERSITY,
SHEFFIELD.

Handwork and Arithmetic.

PART I.

SYNOPSIS.

1. MEASURING IN INCHES.

One of the greatest difficulties in teaching children to measure in inches is for the child himself to grasp that an inch is the *space* between two inch marks. Too much is assumed by the teacher as understood, and the child is allowed to "muddle" through. In order that he may the more easily and permanently assimilate this, it will be well for him to have some

Preliminary Practice.

1. Let him measure how many of his own foot lengths there are in the length of the room, the breadth, etc.

2. Let him "span" the length of the desk, width of the window, cupboard, or other convenient object.

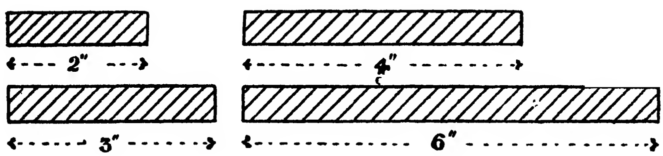


FIG. 1

Strips of Paper.

3. Strips of paper about half an inch wide (Fig. 1), spent matches, match-boxes pulled to pieces, or the Kindergarten sticks, may follow, until the idea is well established that it is the number of lengths of the stick, span, foot, etc., that we require to know in the distance measured.

4. Now follow with a ruler or measuring stick (marked in inches only, for preference) less than 12 in. long. Commence in the following manner :—

Tell the children to draw lines on their papers from mark 1 to mark 5, or from mark 2 to mark 6, in various positions—sloping, level, and upright. Now let a strip of paper or stick be cut, torn, or broken the same length as any one of these lines. With this strip have the other lines measured. Also have them measured with a spent match (about 1¾ in. long), and educe that all the lines are the same length, the only point of difference being their *direction.*

Direction.

It will be well here to spend a little time on this matter of "direction." Objects around the room may be used to exemplify each of the terms—vertical, horizontal, and oblique. The simpler terms used previously may still be employed; the aim largely at the present stage is to bring to the focus of consciousness, through his own self-activity, clear and definite ideas, rather than to burden the young child's mind with difficult words which he

cannot fully understand. "Direction" is of such paramount importance in our entire surroundings, that the fundaments may wisely be given birth to at this stage. The way the pictures hang, the direction the motor-car is travelling, the flow of the river, the slope of the ladder, the spire of the church, are all a question of "direction."

Continuing, the teacher should demonstrate on the blackboard with cotton and pins the truth that "A straight line is the shortest distance between its two ends." The children also should be permitted to do this for themselves and to experiment for themselves. A curve may be made with the cotton, or another pin placed out of the straight while the ruler is placed from pin to pin (Fig. 2).

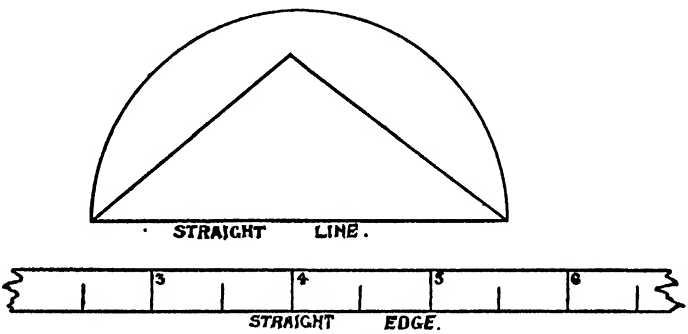


FIG. 2

The children should come to this conclusion without pressure, and they should also state their conclusions in their own words. The same thing may be educed by permitting a child to walk from one point to another in the yard or room, and then to return by a detour; comparison being made of the two routes.

Next tell children to draw lines as before, only from mark 1 to mark 3, or from mark 2 to mark 4. The strips of paper or sticks used in the preceding exercise should now be utilised in this, and the children allowed to discover for themselves that—

1. the lines of the first set are *twice* the length of the second set, and that
2. those of the second set are *half* the length of the first. (See Fig. 1.)

This may be more easily discovered if the children be permitted to fold the long strips across the middle or break the stick into two equal parts. The children should not be *told* what to do, but allowed to follow their own method—within practical limits—of arriving at the truth.

Repeat the same process with lines drawn from mark 1 to 4 and from mark 1 to 7.

Return to the short pieces (2 in. long). Tell children that we

want to find the true length of these. Allow them to tear them in two, and discuss how far they reach on their rulers; the short pieces from one mark to the next, the longer pieces from one mark to the next but one (Fig. 3). Here introduce the word *inch*, and say that the short piece is 1 in. long, the longer piece 2 in. long. Compare these short pieces with the longer piece (4 in.), and deduce the length of that. Have these strips of paper marked up into inches, and allow children to use them for measuring with at this stage.

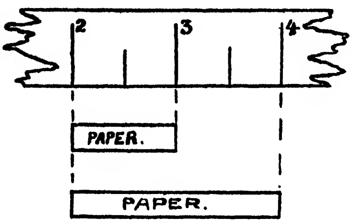


FIG. 3

The 3 in. and 6 in. strips should now be treated similarly, and marked off into inches from the ruler itself or the child's own strip.

The strips of paper used may be of different colours. Problems based on these should be given, and the children allowed to formulate questions for themselves and give the answers.

Suppose that the 1 in. is red, 2 in. green, 3 in. blue, 4 in. yellow, 5 in. brown, 6 in. white.

2. HANDWORK EXERCISES.

Letters, etc.

Numerous other examples will occur to the teacher similar to the following, and with a little practice the children will work

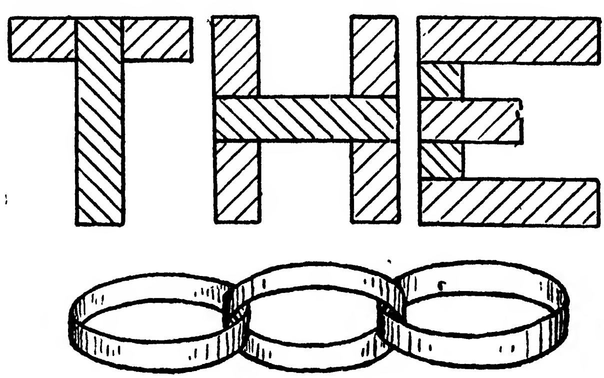


FIG. 4

these very quickly. The strips may be used up in making many simple models similar to those in Fig. 4, giving a practical bearing to the work from the child's point of view.

Other letters can and should be made with these strips: a right angle—letter L; a sharp angle—letter V; and in letter X both

sharp and blunt angles. Various problems can be based on these letters as to how long a strip would be required to make certain letters; how much longer one part is than another; questions as to cost; which strips are equal in length; strips added, subtracted, multiplied, and divided. Always see that the result is obtained *practically* before allowing the children to state the result in words or writing.

Problems and Exercises—I.

1. Find the length of each strip.
2. Find how long red and blue will reach when placed end to end.
3. Mark the middle of the yellow and the white strips, and prove by folding or cutting.
4. Which is the longer, the blue or the brown; and by how much? Prove by superposition.
5. How much is the brown longer than the green? Prove your answer by either superposition, folding, or cutting.
6. How much of a white strip would you have to put on the end of the blue and the green to make them as long as the brown and the red? Do this.
7. If I paid a penny for every inch of strip I bought, how much should I pay for the green, red, blue strip, etc.?
8. A fly walks along the red and the white strips. How far does he walk?
9. If a fly walks 1 in. in 1 min., how long will it take him to walk along the white and the red strips? Etc.
10. If he walks 2 in. in 1 min., how long will it take him to walk along the yellow and the white strips? How far will he walk in 4 min.? Which strip could he walk in 1 min., in 3 min.?
11. If I put five green strips end to end, how far would they reach?
12. How many of the red strips would reach as far as the green, blue, etc.?

Coincidence.

Reference should be made to the previous examples worked with the strips, that when two strips began at the same place and ended at the same place, we said that they were the same length. What is wanted now is to prove satisfactorily to the child that "magnitudes which coincide are equal."

Use various sized squares and oblongs of paper or cardboard, and let each child have a set of each. (See Fig. 5.)

1. Place B on A. Are they the same size? How do you know?
2. Place A on B. Can you see B now? Why? How do you know they are of different sizes?
3. Place C on A. Is it the same size as A? How do you know?
4. Treat others in the same way.
5. Superpose two books, two pennies, etc.

6. Superpose A on A^1, B on B^1, etc. Are they the same size? How do you know?

Here deduce that when two (or more) lines, papers, books, etc.,

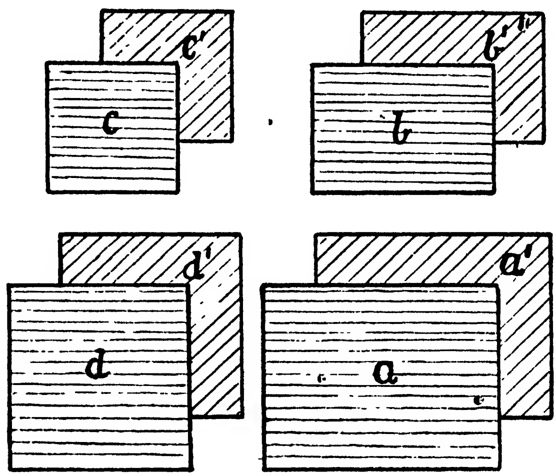


FIG. 5

"exactly fit" on each other, they are the same size. This is a sufficiently clear definition at this stage.

3. PROPERTIES OF A SQUARE.

(*a*) *Four straight sides all the same length.*

1. Let each child have a 4 in. or 5 in. or 6 in. square of paper. Tell them each *is* a square.
2. Measure each side of this square with a strip of paper.
3. Measure again with ruler and obtain answer in inches.
4. Then let the children fold and *prove* the sides are all the same length, because they "exactly fit" along each other.

Folding Exercises (Fig. 6).

Fold side D over on to side B—they "exactly fit," then they are both same length.

Fold side A over on to side C—same conclusion. This proves opposite sides are equal.

Fold D over on to C, diagonally; since these two "exactly fit," then D is same length as C.

Fold D over on to A—same conclusion. Treat all the sides in this manner.

Since A = B, A = D, and A = C, then all are the same length and equal to one another.

(*b*) *Four corners all the same shape—all "square" corners.*

Folding Exercises (Fig. 7).

Fold corner M over on to corner Y—they "exactly fit," then both are *same shape.* Fold N over on to X—same conclusion.

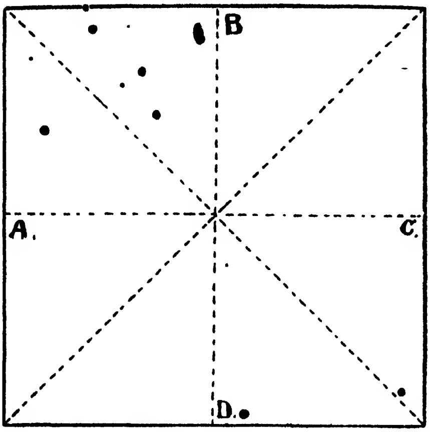


FIG. 6

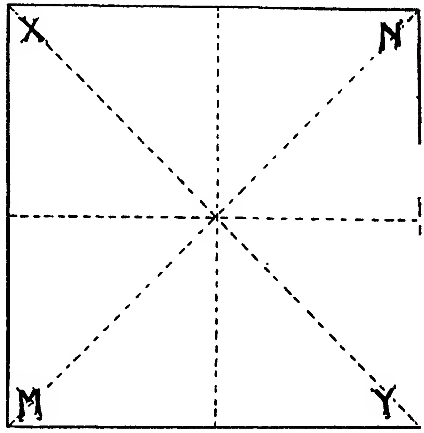


FIG. 7

Fold M over to N, and X over to Y; they exactly fit, then they are *same shape.* In same way fold all the other corners over until all have been superposed.

General Conclusion—

A square has (1) four sides all the same length, and
(2) four corners all the same shape.

Squares of different sizes should be tried by all the children, so that the conclusion may be general and not particular. The same kind of corner should be pointed out by the children, *e.g.*, corners of books, doors, windows, walls, etc. Allow children to invent problems, and give them an opportunity of proving their answer —particularly and then generally.

Problems and Exercises—II.

1. If you have a 4 in. square, how far is it all the way round?
2. How long is the diagonal of a 5-in. square?
3. Place two 4 in. squares side by side and see how far round it is.
4. If I cut a 4 in. square into two equal parts (diametrically), how far would it be round one part?
5. Place these two parts end to end; how far round now?
6. If I cut a 4 in. square into strips 1 in. wide, how far would they reach placed end to end?
7. How far round would it be?
8. If a snail starts at X (Fig. 7) to crawl round the square, how far has he gone by the time he reaches Y, M, and N?

9. If he crawls 1 in. in 1 min., how long does it take him to go all round? To go to N, Y, M, etc.?

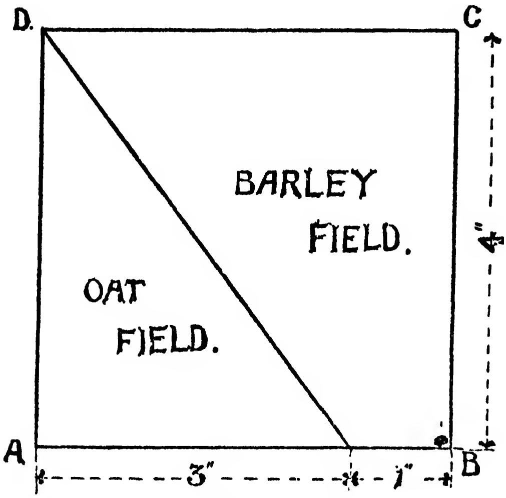


Fig. 8

10. Which is the greater distance, round a 5-in. square or round two 4 in. squares placed side by side?

11. Place a 4 in. and a 5 in. square side by side, and find the distance round.

12. Cut a square, as in Fig. 8, and find out (1) how long DE is; (2) how far round DAE; (3) BCDE; (4) how much farther round BCDE than DAE. (The square may be likened to a field and DE a path across it—the interest in the problems will be increased.)

Axiom I.

Things equal to the same thing are equal to one another.

Supply each child with three 4 in. squares of different colours, *e.g.*, red, blue, green.

Refer to the lesson on "Coincidence," and remind the children of what was learned then. Follow this up by telling children to see if the red and the blue are equal by measuring, folding, and superposing.

Conclusion—they are same size and shape.

Try the green and the blue in the same way.

Now treat the green and the red similarly—same conclusion.

Since red and green are both same size and shape, and since they are each the same size and shape as the blue, we conclude they are *all three* the same size and shape.

Demonstrate also with three boys, books, pennies, etc. Allow children also to give examples, and explain them in the same manner.

4. PROPERTIES OF THE OBLONG.

1. *Four sides—those opposite are same length.*
2. *Four corners—all same shape, all square corners.*

Supply children with various sized oblongs, strips of paper, and rulers.

Tell children to examine these and find out all they can about them on the same lines that they did the square.

Sides.

Use strips of paper to prove that opposite sides are the same length (Fig. 9).

Fold side C over to side D, and A over to B.

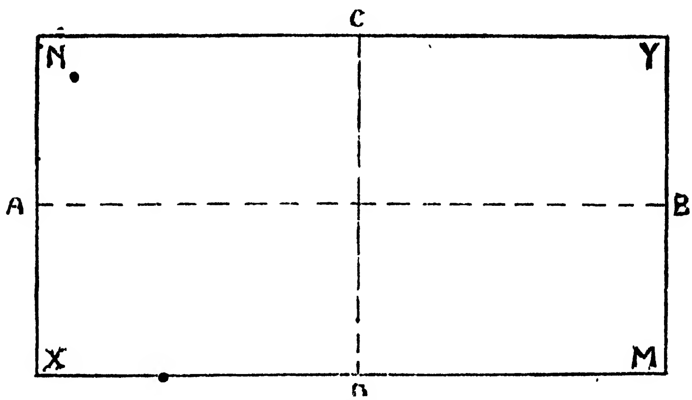


FIG. 9

A same length as B } because they "exactly fit."
C ,, ,, ,, D }

Note that A is *not* the same length as C.

These are the joined sides, not opposite.

Conclusion—*opposite sides are same length.*

Corners.

Fold corner X over on to M } X and Y each "exactly fit" M.
,, ,, Y ,, ,, M }

Then X and Y and M are all the same shape. (Axiom 1.)

Fold corner X over on to N } X and Y each "exactly fit" N.
,, ,, Y ,, ,, N }

Then X and Y and N are all the same shape. (Axiom 1.)

In like manner fold N and M on to X and Y. Also let children use an old square corner—one that has been previously proved in the square—and apply as a template to the four corners of the oblong, proving that all corners are the same shape as the template.

Conclusion—*all corners are same shape.*

Summary—

An *Oblong* has:
1. Four sides, not all same length, but opposite ones equal.
2. Four corners, all same shape and all square corners.

Compare the properties of the square and oblong; note the similarities and differences. Take other oblongs, let children change about, and prove these.

Problems and Exercises—III.

1. How far is it round a 6 in. square ? Cut one into two equal oblongs and find out how far it is round one of these.

2. Cut a 6 in. square into two oblongs in such a way that one is twice the size of the other. Find how far round the large one it is ?

3. Find the distance round the small oblong, and see how much less than the larger one it is.

4. Cut the largest square you can from each of these oblongs. Find how much farther it is round the one than the other.

5. How many squares of the same size as the smaller one can be made from the two remaining oblongs ? Also how many small squares could I cut from the larger one ?

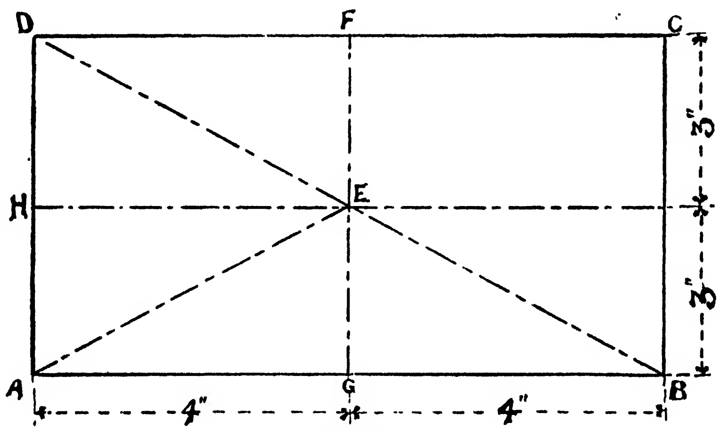


FIG. 10

6. If a fly walks along two long sides and one short side of the small oblong, how far does he walk ? If he walks 2 in. in 1 min., how long does it take him ?

7. Rule up an oblong 8 in. × 6 in., as shown in Fig. 10. Find the length of (1) DE, (2) BE, (3) DB, (4) AE.

8. How far is it : (1) From A to F through D ? (2) From A to F through E ? (3) From A to B through E ? (4) From A to D through E ? (5) From D to B through H and E ? (6) From D to B through A and E ?

5. HALVES AND QUARTERS.

(*a*) Lines and Strips.

1. Tell children to draw lines on their papers 2 in., 4 in., 6 in., and 8 in. long.

2. From long strips given to each child, cut off two pieces, the same length as each of the drawn lines, *i.e.*, 2 in., 4 in., 6 in., and 8 in. long.

3. Find the middle of each strip by folding the two ends together and creasing.
4. In the same way find the middle of each half strip, and crease.
5. Cut one of the half strips into two ; now arrange as in Fig. 11 in exercise book or on paper, and paste in position.

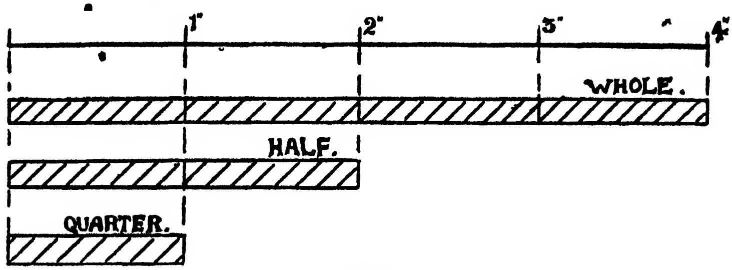


FIG. 11

6. Let children see that there are two halves or four quarters to the whole line. Measure each of the strips so divided, and mark the lines, according to their corresponding parts of strips, into halves and quarters.
7. Question freely on these parts when the practical work is accomplished.

Problems and Exercises—IV.

1. If a line 4 in. long stands for 1d., draw lines to represent $\frac{1}{2}$d., $\frac{1}{4}$d., $1\frac{1}{2}$d., and 2d.
2. An 8 in. strip represents a mile ; cut off pieces to represent $1\frac{1}{2}$ miles, $\frac{1}{2}$ mile, $\frac{3}{4}$ mile, and $\frac{1}{4}$ mile.
3. I pay 1d. an inch for strip, how many inches can I get for 3d., 5d., and 6d. ? Show this in your books in line and strip.
4. A man rode 2 miles by tram, 4 miles by train, and walked 3 miles. If 1 in. stands for a mile, show in strip how far he went. How much farther did he ride than walk ?
5. If a train has eleven coaches and an engine on, and two coaches are represented by 1 in., draw a line which will represent the whole train if the engine is as long as two coaches.
6. A line 4 in. long represents a shilling ; draw lines which will represent 6d., 9d., and 3d. How long will the last line be ?
7. If half a line is 3 in., what is the length of the line ? If a quarter of a strip is 2 in., how long is the strip ? How much is half of it ?
8. If a strip 6 in. long represents the height of a house, and the house is 24 ft. high, how much does 1 in. represent ?

(*b*) Square and Oblong.

(1) *Cutting.*

1. Supply each child with a red and a blue tinted 4 in. square.

2. Tell children to prove that the two squares supplied are both the same size, and ask for their reasons for their conclusions.

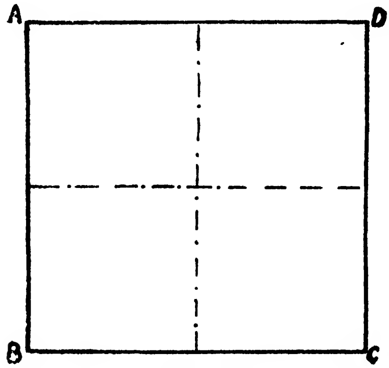


FIG. 12

3. Tell them to "line in" a 4 in. square in their drawing books (ruled in 1 in. squares). Also find the middle of each side of the red square; each side being now divided into halves (Fig. 12). Draw the diameters, and question. Ask also *how* to find the middle of AB.

4. Draw the diameters of this red square and cut along one of them (Fig. 13). Here are two oblongs, each half of the original square. Prove these two oblongs are equal.

5. Superpose the two oblongs and see that they "exactly fit" the blue square. Here are two halves (the oblongs) exactly equal to the whole (square) (Fig. 13).

6. Examine the oblong, corners and sides; describe it. Find the middle of each long side, and so divide each oblong into its halves.

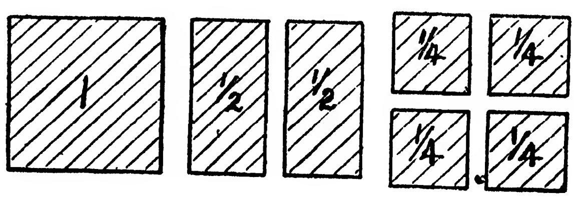


FIG. 13

7. Superpose these halves (squares), and observe the two halves "exactly fit" the original whole (oblong) (Fig. 13).

8. Superpose the four quarters (squares), and notice that they "exactly fit" the original blue square. Paste these halves and quarters in an exercise book, or on paper underneath the drawn square. Test the halves and quarters also on the drawn square, and see that they coincide.

1 whole = 2 halves = 4 quarters.

9. Prove by measuring and folding that the quarters are squares. Compare with the original square also.

(2) *Folding.*

1. Fold the blue square as shown. (Fig. 14.)

2. The relative positions of the various stages in the drawings must be observed. In No. 2 (Fig. 14) it will be seen that the bottom half of No. 1 has gone away. It must have been folded either in front or behind, and as either of these folds would give the same appearance in the drawing, it is marked F to indicate that the fold has been made forward. In No. 3 the same thing applies. In No. 4 the square has been opened out, and the dotted lines indicate the creases.

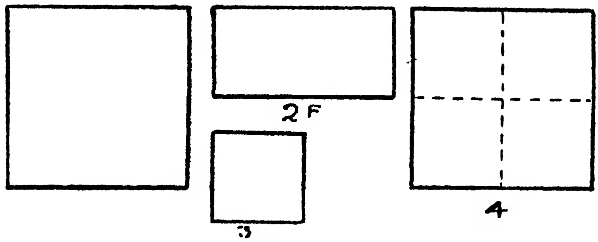


FIG. 14

3. Place the red quarters on top of the folded blue square and verify again. They "exactly fit"; then

4 quarters = 1 whole.
2 halves = 1 whole.
2 ,, = 4 quarters.

(3) *Cutting.*

1. Revise halves and quarters already done, and let the children have fresh red and blue papers, 4 in. square.

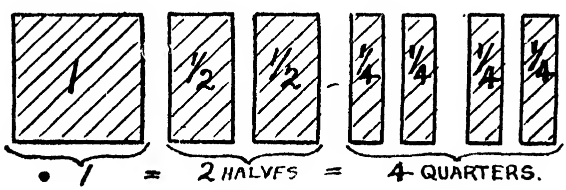


FIG. 15

2. Rule and cut the red one into halves, as in previous exercise.
3. Elicit that to obtain quarters the halves were halved.
4. Suggest another way of obtaining quarters from the oblong half, and permit the children to try for themselves (Fig. 15). The square may be either ruled up and cut, or folded and cut; but greater accuracy can be obtained in cutting to a line than a crease.
5. Prove that the four strips obtained are all the same size by superposition.
6. Prove that two strips exactly fit a half and that the four strips exactly fit the whole.
7. Prove also that the oblong quarter is equal to the square quarter (Fig. 16 *b*) and that the square quarter is equal to the oblong quarter (Fig. 16 *a*) by cutting each across the middle (as

in Fig. 16) and reconstructing. Test with a whole quarter in each case.

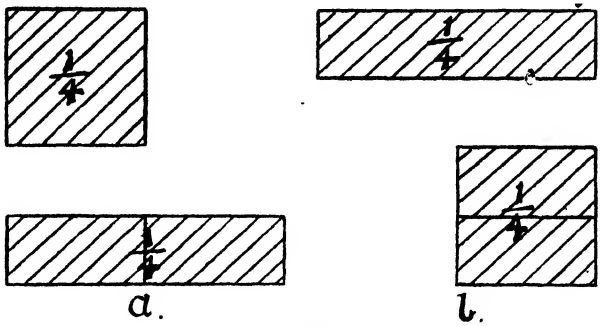


FIG. 16

Same conclusion as previous exercise.

(4) *Folding.*

1. Fold the blue square as shown in Fig. 17, and note the relative positions of drawings.

2. Open out the paper and compare with the cut strips; superpose as before, and see that they all exactly fit.

3. Encourage children to use both hands in folding, alternately left and right, remembering that it is essential to develop the child's brain through his *hands*, not through *one* hand for one particular fold. This might do for a mechanic compelled to work under a system of "division of labour," but will not tend to that general development which is desired in a child of tender years.

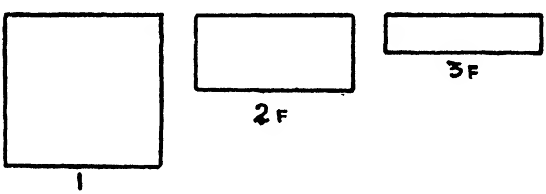


FIG. 17

(5) *Cutting.*

1. Supply each child with 4 in. red and blue squares of paper.

2. Suggest another way of halving the square besides the way already done, cutting along the diameter. Let children try for themselves.

3. Learn the name diagonal, and cut along one of them after being drawn or folded; preferably the former (Fig. 18).

4. Superpose each of the halves, and show that they are both the same size. See that these two halves exactly fit the blue square.

5. Compare the oblong half and the triangular half. Place the

oblong half as in Fig. 19, and cut off the shaded portion of the triangular half. Place it in its new position on the uncovered piece, and prove that they are equal.

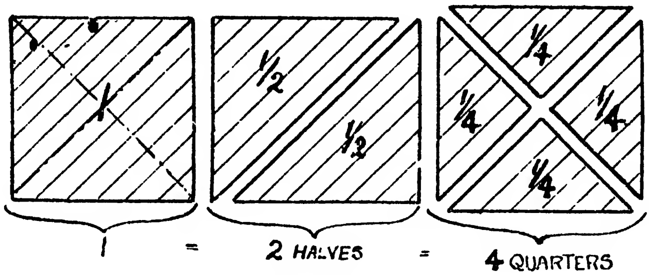


FIG. 18

6. Resolve the oblong half into a triangular half, and compare with a whole one. Fold down top corner (as in Fig. 20 (*b*)). Cut this off along the crease, and paste in exercise book, as shown in Fig. 20 (*c*). Verify with a whole triangular half.

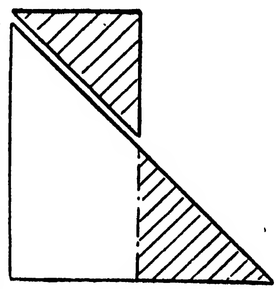


FIG. 19

7. Deal with quarters in the same way. To obtain the new shaped quarters, we again "halve the halves." To do this, allow the children either to cut along the diagonal, or fold and crease, and cut along that.

8. Superpose the red quarters on the blue square and the two halves.

9. Tell children to resolve the oblong half and the triangular half each into a square, and compare (Fig. 21). Test with the original quarter square (Fig. 13).

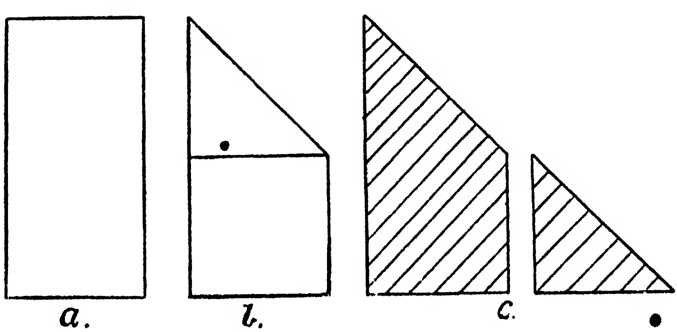


FIG. 20

Summary. The square quarter, the oblong quarter, and the triangular quarter are all the same size, though not the same shape.

10. Unless children are already familiar with the word triangle,

it would be well to use the term "three-cornered figure" until they grasp the meaning.

11. It will be seen that many opportunities occur to exemplify Axiom 1, and this should be required of the children in their simple steps of reasoning.

12. In the same way, halves of the same thing (or of equal things) are equal: as 6 pennies, 2 threepenny pieces, and 1 sixpence are equal to one another, because they are all half of a shilling.

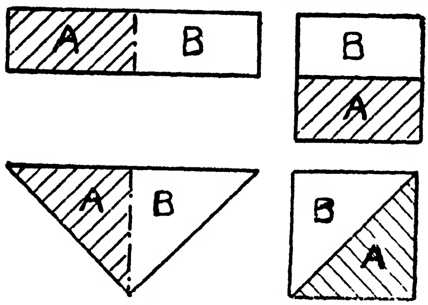


FIG. 21

(6) *Folding.*

1. Fold the blue square, as shown in Fig. 22, and treat in the same way as in Figs. 14 and 17.

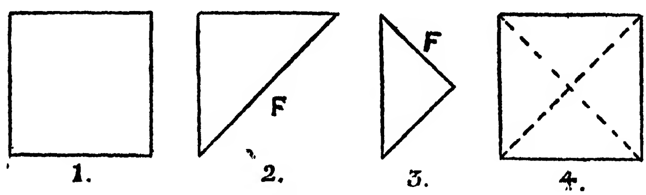


FIG. 22

2. Superpose cut parts, and verify as in previous cases.

(*c*) **The Oblong** (*continued*).

1. Supply each child with a paper oblong 6 in. × 8 in. Draw lines as shown (Fig. 23), and test their length.

2. Cut along the diagonal, and prove that the two triangles are equal by superposition and measuring. Prove that each is half of the original.

(1) *Cutting.*

1. Cut out the shaded triangle in Fig. 23, and reconstruct as in Fig. 24. The figure is oblong now and equal to the triangle.

2. Cut out the corresponding piece in the other triangular half, build up in the same way, and place side by side. Compare with an oblong similar to the original.

3. Cut up the original oblong into triangles, the same size as shaded one in Fig. 23, and see how many are required to form the oblong 6 in. × 8 in.

4. Build up various figures with these triangles—triangles, oblongs, diamonds, etc.

5. Make a "diamond" with four small triangles; find the distance round and across each way. Find how far each corner is from the centre.

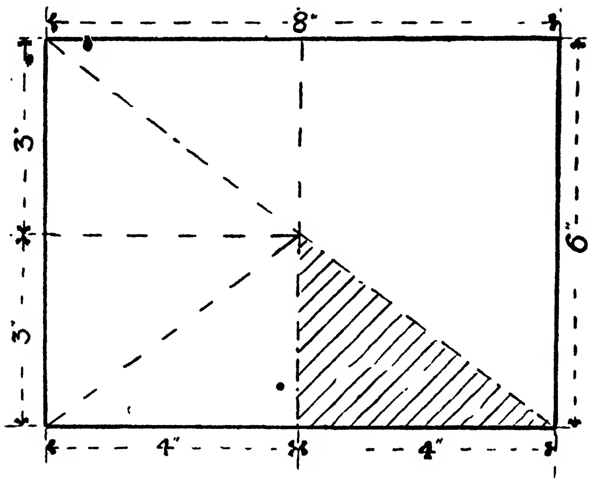


FIG. 23

6. Make a flag (6 in. × 8 in. paper), as in Fig. 25, by cutting out shaded triangle; cut shaded triangle into two halves, as shown; superpose on A and B, and prove XY is one quarter of

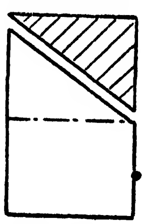


FIG. 24

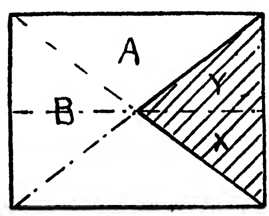


FIG. 25

original oblong. Note different shape of A and B, but still same size, because they exactly fit.

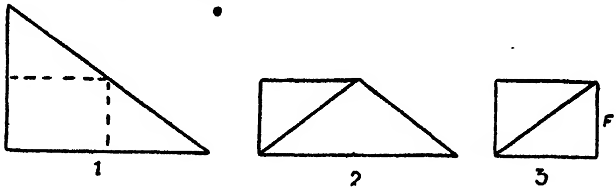


FIG. 26

(2) *Folding.*

1. Take a triangular half of a 6 in. × 8 in. oblong and fold as in Fig. 26; try No. 3 (Fig. 26) on another oblong, and notice

that it takes four such to completely cover it. Since No. 3 is double in thickness, it means that two single triangles will cover the oblong. Cut up No. 1 and compare with No. 3.

2. By superposing, see how many shaded triangles (Fig. 23) may be cut from half the oblong.

6. FURTHER PRACTICE WITH SQUARE AND OBLONG.

1. The figures already cut will afford numerous opportunities for further practice in the making of problems and exercises. Allow children to make them for themselves, and utilise the best as class exercises.

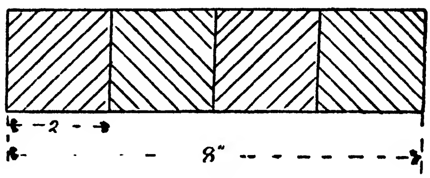


Fig. 27

Take, as example, the 4 in. square cut into four 2 in. squares. Place in various positions, and obtain different results (Figs. 27 and 28). Sums should be made simpler at first by using only two or three squares, and regarding them as fields or gardens with paths across; plots joined together, etc.; in fact, anything that will make the work real, alive, and interesting to the children. Verify results by measurement.

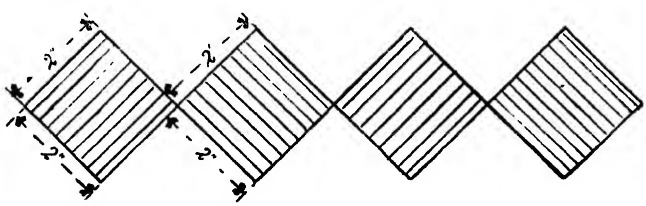


Fig. 28

Set down on paper :—

Fig. 27.

8 in. long side
2 ,, short ,,
8 ,, long ,,
2 ,, short ,,
—
20 ,, all round.

Fig. 28.

8 × 2 = 16 in. top side
8 × 2 = 16 ,, bottom side
—
32 ,, all round.

2. Take an oblong 4 in., × 1 in. test, and prove that same is an oblong by measuring and folding as before.

Find length of sides, distance round (Fig. 29 B); place two together, and obtain another oblong (Fig. 29 A). Find its perimeter.

How much is A larger than B? Prove by superposition.

Divide A diagonally; demonstrate that each half is equal to B by reconstruction, folding, etc.

Build up, with these squares and oblongs, various shapes and figures (Fig. 30); encourage originality and inventiveness of children. Make sums on these in addition and subtraction. Place strips (as in Fig. 30) to form oblong and square. Compare these figures, sides, corners, lengths, etc.

Exercises in estimating lengths should also form a part in these examples. Having estimated a distance, it should be noted down in a table similar to Fig. 31; test by measurement. Note the amount of error, and compare results.

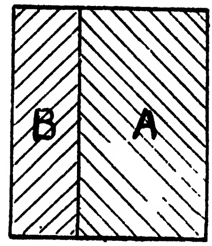


Fig. 29

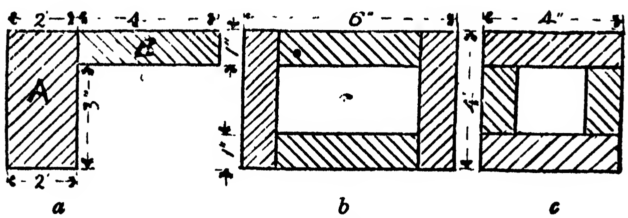


Fig. 30

3. Again, take a 4 in. square and divide up into two squares and two oblongs (Fig. 32). Prove what figures these are by investigating; compare the two squares, the two oblongs, and interchange. Divide A into quarters by folding, and C and D into halves.

Estimate.	Actual Distance.	Error.
6 in.	5 in.	1 in. too much
10 ,,	11 ,,	1 ,, ,, little
7 ,,	9 ,,	2 ,, ,, ,,
4 ,,	6 ,,	2 ,, ,, ,,

Fig. 31

Problems and Exercises—V.

1. If small square B cost 1s., how much would large square A cost? (Fig. 32). Also C?

2. If square A were cloth sufficient to make three suits of clothes, cut a piece to show the size I should want for one suit.

3. If I can make a jacket from C, how many jackets can I make from A and D together?

4. How many tiles the size of B shall I want to cover a floor as large as A? Also how many would be required to cover (*b*) in Fig. 30.

5. Supposing it is 12 miles round A (Fig. 32), how long is one side of the square?

6. How much longer is C than it is wide? Find the distance round it, and see how far D and C, placed end to end, will reach.

7. What is the difference between the distances round A and D?

8. Tom runs round C and Jack round A; which runs the farthest, and by how much? (Suppose 1 in. = 1 mile.)

9. How many times will Tom have to run round B to run as far as George, who runs round D?

10. How long will Jack take to run round B, if he runs 1 mile in 5 min.? (Suppose 1 in. = 1 mile.) Similarly Tom round A?

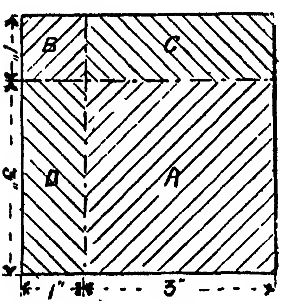


FIG. 32

11. Divide a 4 in. square into two parts, such that one part is three times the size of the other.

12. Find out how far it is round each, and how much farther round one than the other.

13. Arrange a number of small squares in as many different ways as you can to form patterns for floors, borders, etc.

14. Use the small and large squares and oblongs to make further designs.

15. If a triangle and an oblong are each half a square, are they always the same distance round?

16. Make a triangle from an oblong, by dividing the oblong into two equal parts.

17. Cut up a square in such a way as to make a four-sided figure which is not a square. Cut an oblong similarly.

18. From an oblong 6 in. × 3 in., cut diagonally and diametrically longways (draw these first), and resolve into two squares.

19. In Fig. 30 (*c*) and (*b*) how much of the whole figure is each inside square and oblong?

20. In Fig. 30 (*a*) how many of B will be required to make A? How many times B is A?

21. How much farther round is the oblong than the square in Fig. 30? Inside the same.

22. From a 4 in. square make an oblong and a triangle which shall each be half the square. Prove by superposition, cutting, and folding.

7. MEASURING HALF INCH AND QUARTER INCH.

1. Supply children with papers ruled[1] with lines of various

[1] *Note.*—These may be cyclostyled or graphed.

lengths, in inches and half inches. For convenience, have them numbered and drawn in various positions—level, upright, and slanting.

Tell children to measure one particular line (*e.g.*, $3\frac{1}{2}$ in., as in Fig. 33), and find its length. It will be seen to be more than a whole number of inches—a " bit over " or to spare.

2. Direct attention to this " bit over," having marked the number of whole inches on the line (Fig. 33). Have a strip of

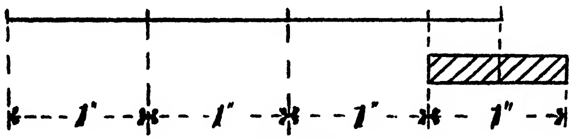


FIG. 33

paper cut an inch long; mark off on this strip the length of the " bit over." Fold strip across, and notice crease coincides with mark. Knowing from previous exercises that these two portions are halves, then the " bit over " must be half of the strip; and since the strip is 1 in. long, then the " bit over " must be half an inch.

Then the line is $3\frac{1}{2}$ in. long.

3. Proceed with others in the same way, and have the length of line as found by each child written underneath each line.

4. Have some estimation work done with some of the lines, and afterwards tested, and the amount of error in each case noted, as done before and shown in Fig. 31. Further estimation should be carried on with familiar objects in the same way as above.

5. Tell children to draw other lines in certain directions a certain length, incorporating inches and half inches. Have objects pointed out in room according to their direction; pencils or pens held in certain directions,

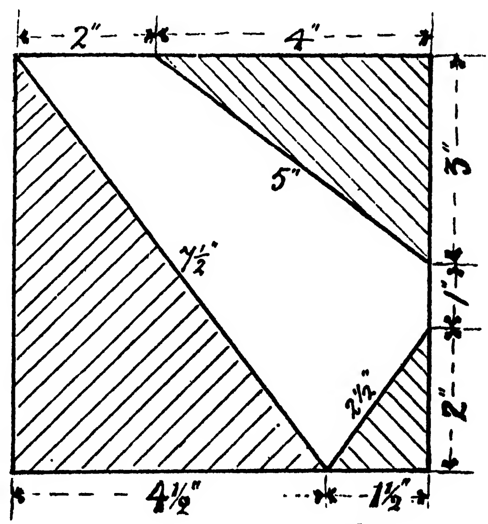


FIG. 34

papers, books, etc. Combine pencils and papers—touching each other and in certain directions, *e.g.*, place book upright; place pencil sloping and book level, etc.

6. Take a 6 in. square, mark off the dimensions on the sides, as shown in Fig. 34, and draw the lines. Tell children to estimate lengths of drawn lines; measure them, and note amount of error. Cut along these drawn lines, and measure again.

Problems and Exercises—VI.

1. Find the distance round each triangle (Fig. 34).
2. Superpose smallest triangle on medium one, and find how many of the former are required to make the latter.
3. Find how far it is round the irregular piece.
4. Draw straight lines equal to the distances round each of the two smaller triangles.
5. How much farther is it round the largest than the medium? The medium than the smallest?
6. How many times should I have to run round the smallest to go as far as round the medium? As far as round the largest?
7. If I went round the largest twice, how many times should I have to go round the smallest to go as far? Round the medium to go as far?
8. Draw a line half as long as the distance round the largest triangle, and another half as long as the distance round the irregular figure.

THE QUARTER INCH *should be taught in the same manner as the half inch has been done in the foregoing paragraph.*

After this further exercises (Fig. 35) should come, to give

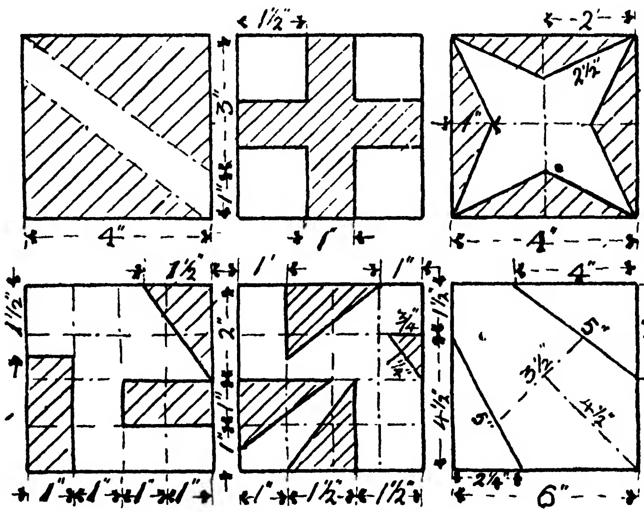


FIG. 35

practice in measuring in inches, half inches, and quarter inches, and sums based on them as in the former exercises. Have the

various figures drawn and cut out, and mounted either on other tinted papers or in books.

8. PROOF OF MULTIPLICATION AND DIVISION.

Supply each child with a 4 in. square of thin cardboard or paper. Divide each side into inches, and rule up into sixteen inch squares. Cut this square into four strips 4 in. × 1 in., and each strip into four squares 1 in. × 1 in.

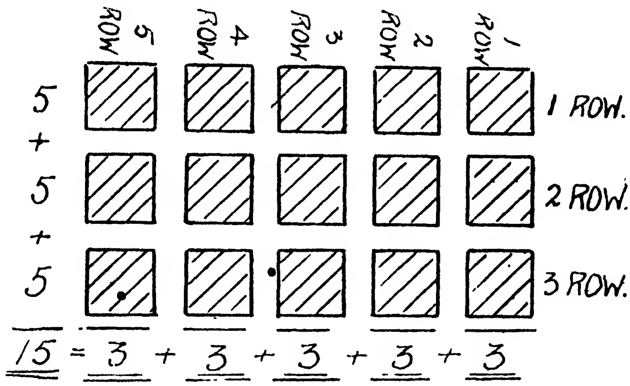


FIG. 36

(1) *To prove that* 3 × 5 = 5 × 3 = 15 (Fig. 36).

Take fifteen of the squares and arrange in rows of five, *i.e.*, three rows; again there are five piles, and three in each pile. There are fifteen in all, and so we see that

3 rows of 5 each = 15.
5 piles of 3 each = 15.

(2) *To prove that* 16 ÷ 3 = 5 *and* 1 *left over* (Fig. 37).

Take sixteen of the squares and arrange in piles of three each, as far as they will go. On counting up, we find five piles of three in each and one left over, *i.e.*, there are 5 threes in sixteen and one left. If larger numbers are required, larger pieces of cardboard must be given to the children to divide up.

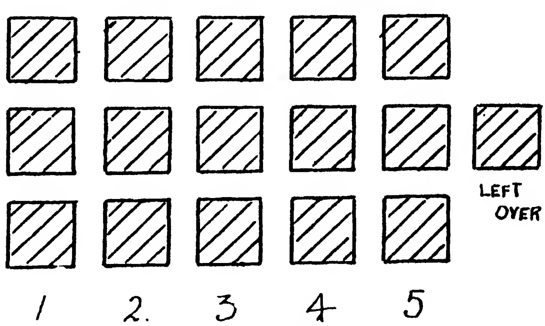


FIG. 37

9. HANDWORK EXERCISES.

(1) Illustrative of the Half.

See-Saw (Fig. 38). Two strips of paper 4 in. × 1 in. required.

Fold one piece across the middle for the support, and into quarters the opposite way to form the feet, as shown. Balance the other strip, and measure to verify the middle.

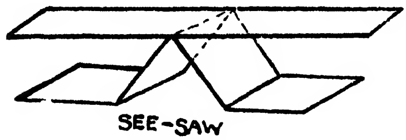


FIG. 38

Tent and Note-Book (Fig. 39). Square or oblong pieces may be used according to shape of note-book required. It is a good plan to have thin white paper for the leaves and a tinted paper for the cover. These should be stitched together and tied inside.

(2) Illustrative of the Quarter.

Admiral's Hat (Fig. 40). Any size square paper may be used. Draw the diagonals; cut out the shaded portion in 1. Fold right quarter up on to top quarter, 2; fold left quarter up on top ditto, F indicating the forward fold. Paste down the edges.

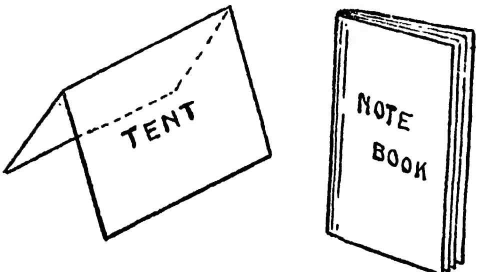


FIG. 39

Windmill (Fig. 41). Use a 6 in. square. Draw the diagonals and cut along about one-third of the distance from each corner. Turn the alternate corners over to the middle and paste them down. If a piece of cardboard be used underneath the centre of the square, it strengthens it to take the pin to fasten on to the stick.

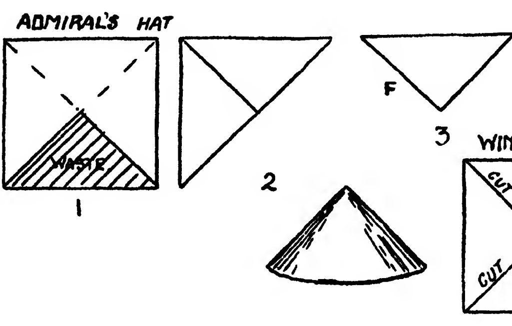


FIG. 40

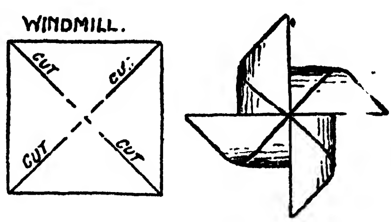


FIG. 41

Screen (Fig. 42). Any oblong paper will do. Notice that the middle crease is a "crest" crease indicated by the irregular dotted line, while the "furrow" creases for the two side wings are

indicated by ordinary dotted lines. This indication is adopted throughout.

Let children ornament the top of each wing as they choose; but let them all be the same. The diagram shows four methods.

Some embellishment may with advantage be added either by pencil or brush.

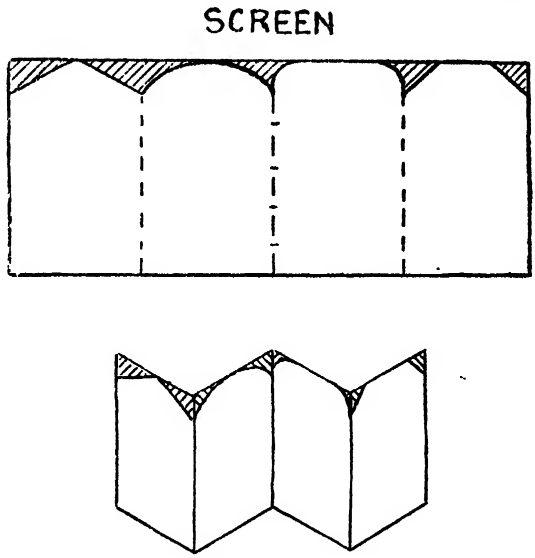


FIG. 42

Knot Pattern Note (Fig. 43). Any oblong paper will do, but about the size of ordinary note will be best. In Nos. 2 and 3 the F indicates the forward fold. Crease No. 3 as shown. For No. 4, note its position in regard to No. 3. No. 5 would best be seen if drawn immediately underneath No. 4. Allow children to address the note; if thought desirable, a note may be actually written previous to this exercise, allowance being made for the age of the pupils.

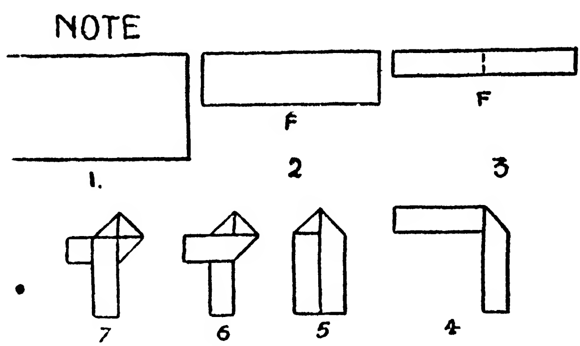


FIG. 43

Frequent opportunities will occur during these lessons to link up the arithmetic with the handwork. The inter-relation of one drawing with another will give variation in examples of measurement, estimation, size, etc.

Problems and Exercises—VII.

1. In Fig. 38, if the bottom strip is divided into four equal parts, how long is each part? How much of the whole is it?

2. Resolve the waste piece in Fig. 40 into an oblong. How many

pieces does it have to be cut into ? How many to make it into a square ?

3. What portion of the whole square is the size of the hat in Fig. 40 ? How much of the whole square is used in construction of the hat ?

4. If the screen is 6 in. long, how wide is each wing ? Place the screen so that each wing makes a "square" corner with the next one.

5. How much of No. 1 is No. 2 in Fig. 43 ? No. 3 of No. 2 and No. 1 ? If No. 1 is 5 in. wide and 9 in. long, how wide are No. 2 and No. 3 ? How far round each ? More round No. 1 than No. 2 ? More round No. 2 than No. 3 ?

6. Explain what has happened in Fig. 43 to make No. 5 from No. 4; No. 6 from No. 5; and No. 7 from No. 6. Why is it necessary to crease the middle of No. 3 ?

10. SIMPLE FRACTIONS.

As preparatory work to more difficult problems in addition and subtraction of fractions, the following exercises may be given now or combined with previous examples in halves and quarters :—

(1) **Equal Parts.**

(*a*) a whole = 2 halves = 4 quarters.

(*b*) $1 = \frac{2}{2} = \frac{4}{4}$.

Tell children to draw a line 4 in. long and divide it into inches. Show by longer marks (Fig. 44) that the whole line is divided into two halves, and that each half is again divided into halves by other and shorter marks, giving quarters.

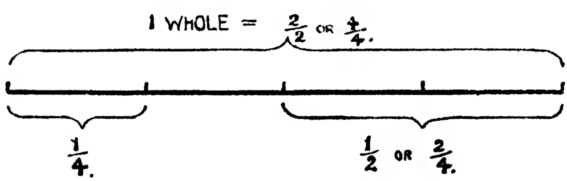


FIG. 44

Have strips of paper cut 6 in. long, and find halves and quarters by folding. Note that the half contains two quarters and that the whole contains four quarters.

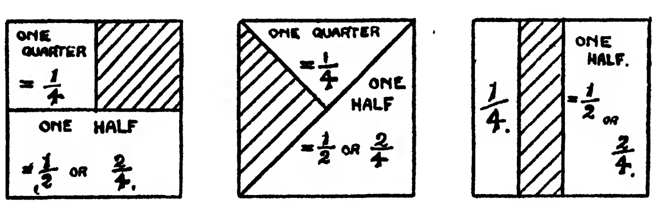


FIG. 45

Repeat the same with a 4 in. square of paper. Let children fold into halves and quarters, and write the names of the various parts

on each, as in Fig. 45. Cut each into halves, and one half into its two quarters. Superpose the two halves on another 4 in. square and the two quarters on the uncut half, and learn—

2 quarters = 1 half, or $\frac{2}{4} = \frac{1}{2}$,
4 ,, = 1 whole, or $\frac{4}{4} = 1$,
1 ,, = 1 fourth, or $\frac{1}{4} = \frac{1}{4}$.

Let children see that $\frac{1}{2}$ and $\frac{1}{4}$ are short forms of stating the respective fractions in words.

(2) To Add and Subtract.

(*a*) $\frac{1}{2} + \frac{1}{4}$.

Tell children to draw a 4 in. line and divide it into halves and quarters. Mark off a half and an adjoining quarter, as shown in the diagram (Fig. 46). Revise the number of halves and quarters in the whole, and show that $\frac{1}{2}$ (or $\frac{2}{4}$) + $\frac{1}{4}$ of line = $\frac{3}{4}$ of line. Repeat with strip.

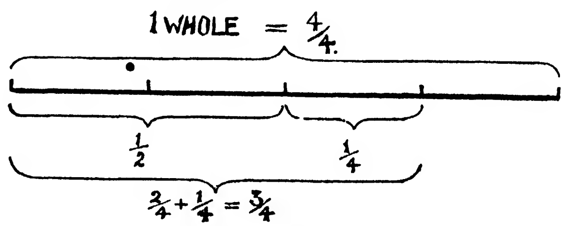


FIG. 46

Take the same example with a 4 in. square. (Fig. 47.) Fold into quarters, and mark off two of these (or $\frac{1}{2}$) and one other. Notice that $\frac{3}{4}$ of the whole square is now taken up and that one quarter remains.

(*b*) $1 - \frac{3}{4}$.

Refer to the line (Fig. 46) and square (Fig. 47), and let children cut away $\frac{3}{4}$ of strip worked previously; also the square similarly. These two exercises may be worked concurrently. Notice what fraction of the whole remains; superpose or measure, as the case may be, and note this down—

$$1 - \frac{3}{4}$$
$$= \frac{4}{4} - \frac{3}{4}$$
$$= \frac{1}{4}.$$

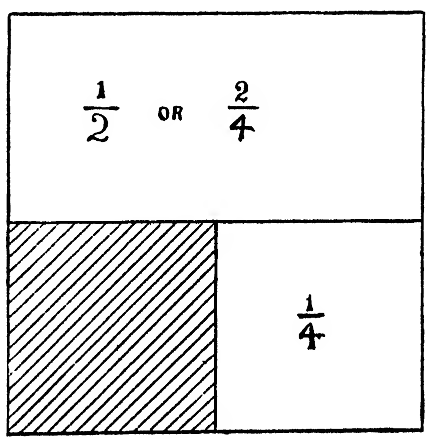


FIG. 47

(*c*) $2\frac{1}{2} - \frac{3}{4}$.

Rule or fold two and a half whole squares into quarters, as in Fig. 48. Cut out $\frac{3}{4}$ of another square. An amount equivalent to this latter portion has to be subtracted from the former. In the $2\frac{1}{2}$ units there are by conversion found to be 10 quarters ($\frac{10}{4}$). From this $\frac{3}{4}$ have

to be taken away. Set down in the following manner, and have the sum worked this far :—

$$2\tfrac{1}{2} - \tfrac{3}{4} = \tfrac{10}{4} - \tfrac{3}{4}$$
$$= \tfrac{7}{4}.$$

The answer $\frac{7}{4}$ will be seen to be more than a whole unit ($\because \frac{4}{4} = 1$). Then cut away from the $2\frac{1}{2}$ units an amount $= \frac{3}{4}$ and notice what fraction remains; it will be seen to be $\frac{7}{4}$ or 1 whole unit and $\frac{3}{4}$ of another. Then $\frac{7}{4} = 1\frac{3}{4}$.

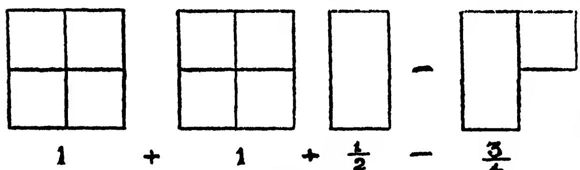


FIG. 48

(*d*) $3\frac{1}{4} + 2\frac{1}{2}$. Fold up several squares into quarters, etc., as indicated in Fig. 49. Set out $3\frac{1}{4}$ and $2\frac{1}{2}$ squares, and note that there are whole units and fractions to be added :—

$$3\tfrac{1}{4} + 2\tfrac{1}{2} = 3 + 2 + \tfrac{1}{4} + \tfrac{1}{2}$$
$$= 5 + \tfrac{1}{4} + \tfrac{2}{4}$$
$$= 5\tfrac{3}{4}.$$

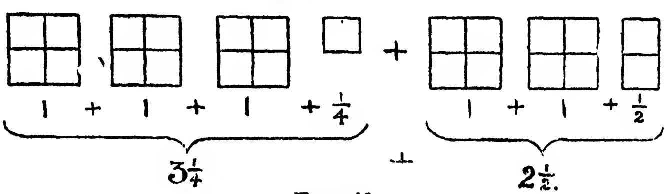


FIG. 49

Repeat the same with lines and strips. Reverse the exercise, and take it as a subtraction sum similar to (*b*) and (*c*), viz. :—

$$3\tfrac{1}{4} - 2\tfrac{1}{2} = \tfrac{13}{4} - \tfrac{10}{4}$$
$$= \tfrac{3}{4}.$$

Problems and Exercises—VIII.

1. If Tom had $\frac{3}{4}$ of a shilling and gave away 6d., how much of a shilling had he left ?
2. A man had $3\frac{1}{2}$ sovereigns; he earned $2\frac{3}{4}$ more; how much had he then ?
3. In one pocket I had $6\frac{1}{2}$ pence, in another $3\frac{1}{4}$ pence, and in another $\frac{3}{4}$d.; how much had I altogether ?
4. To B—— it is $5\frac{1}{2}$ miles; I rode $4\frac{3}{4}$ miles and walked the remainder; how far did I walk ?
5. How many halves and quarters are there in $2\frac{1}{2}$, $4\frac{1}{2}$, 16, $17\frac{1}{2}$?
6. Find the number of quarters in $3\frac{1}{4}$, $5\frac{3}{4}$, $7\frac{1}{2}$, and $18\frac{1}{4}$?
7. How many whole units are there in $\frac{5}{2}$, $\frac{7}{4}$, $\frac{11}{2}$, $\frac{19}{4}$, $\frac{43}{2}$, and $\frac{61}{4}$?

PART II.

SYNOPSIS.

1. EIGHTHS.

(1) Lines and Strips.

1. Tell children to draw a line 4 in. long; find the middle, and then divide the line into quarters (Fig. 1).
2. Cut off a strip of paper (about $\frac{1}{4}$ in. wide) the same length as the line; fold it in halves and quarters as previously.
3. Cut the strip up into quarters; fold all of them across the middle; see how far they will extend along the line when placed end to end.
4. Cut all the quarter strips across the crease mark; place end to end along the line. Notice that there are eight—double of the number of quarters.

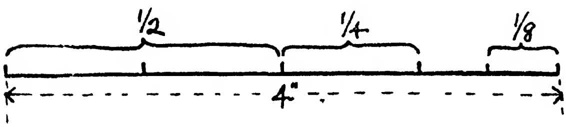


FIG. 1.

5. Give the name of these portions; elicit why so called.
6. Observe a ruler and the divisions of any 1 in. Compare this with line in Fig. 1. Name the small divisions shown—$\frac{1}{8}$ in.
7. Tell children to draw lines on their papers of certain lengths, including all the parts of the inch already mastered, viz., half inch, quarter inch, and eighth inch.
8. Have lines drawn at random, any length. Divide them into any number of parts, haphazard, say, 2, 3, 4, or 5. Have each part measured and its length set down; add up the parts and verify by measuring the whole line.
9. Draw a line, say, 10 in. long; guess the middle: test by measuring. Do the same with quarters and eighths. Test also by cutting strips the same length, and folding and cutting.

(2) Square.

1. Supply each child with two 4-in. squares of paper. Let each child divide one of the squares into quarters diametrically,

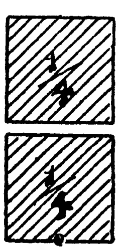


FIG. 2

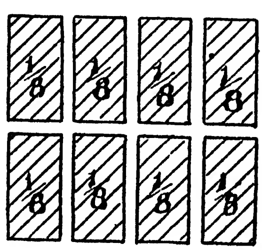


FIG. 3

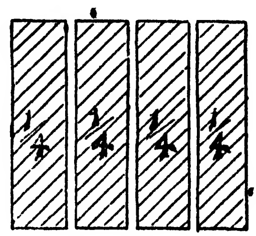


FIG. 4

i.e., four square quarters (Fig. 2). The diameters may be ruled and cut, or creased and cut, but preferably the former.

2. Have the other square divided into quarters in any other way but squares (Figs. 4 and 6).

3. These must be halved again by folding, in order to produce eighths (Figs. 5 and 7).

4. Have each eighth superposed on each other to prove they

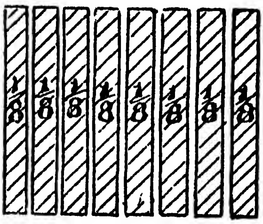


FIG. 5

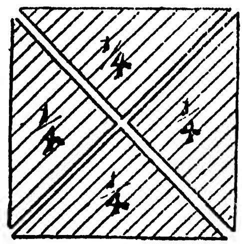


FIG. 6

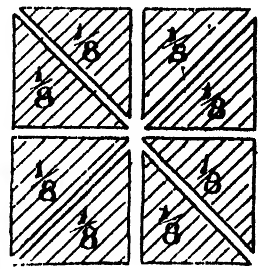


FIG. 7

are all same size and shape. Have any two eighths fitted on to a known quarter (*e.g.*, Fig. 2), and show that two eighths are same size as one quarter.

5. Reconstruct any two eighths, and superpose on one of Fig. 2 for confirmation. Paste each shaped quarter in book, and underneath one of the same shaped eighths. Under this, place two of the reconstructed eighths, making a quarter, of the same shape.

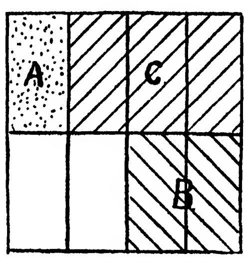


FIG. 8

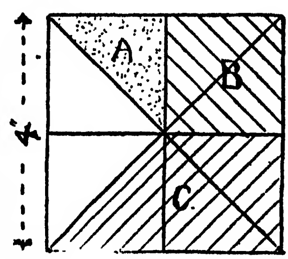


FIG. 9

6. Question frequently while work is in progress.

7. Build up the two squares as shown in Figs. 8 and 9, with different coloured pieces of paper, after having divided whole squares in the necessary manner. Write down the value of the various parts in each case.

2. HANDWORK EXERCISES.

(1) Illustrative of the Eighth.

Church (Fig. 10). Use a 4 in. square of paper. Rule up into oblong eighths, as shown (or fold); measure an inch high for the door and half an inch on either side of the top of this line. Cut out the shaded portion, and leave the "spire"—which involves sixteenths—till the latter are dealt with. Make the necessary cuts for the door.

Seat (Fig. 11). Use an oblong 4 in. × 2 in., 6 in. × 3 in., or 8 in. × 4 in., and stiff brown paper. Rule up, as shown; cut out the shaded portions, fold the legs downwards and the back upwards. See that seat and back and seat and legs form square corners.

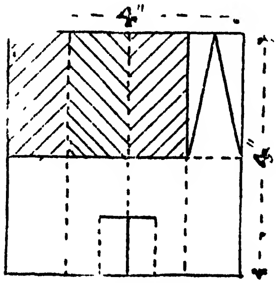


Fig. 10

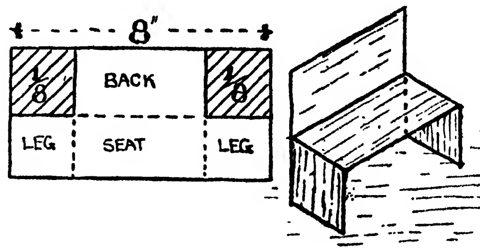


Fig. 11

Sixteenths *should follow in due course, when ample exercise has been afforded to permit of the eighth being thoroughly grasped and understood. It should be dealt with on the same lines as preceding exercises.*

(2) **Illustrative of the Sixteenth.**

Crown (Fig. 12). Use paper 4 in. × 2 in. or 6 in. × 3 in. Divide each short side into two equal parts and each long side into four. Join up these points. Rule in the slanting lines forming the crown and cut out shaded portions. Paste in book.

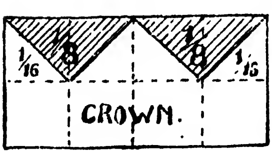


Fig. 12

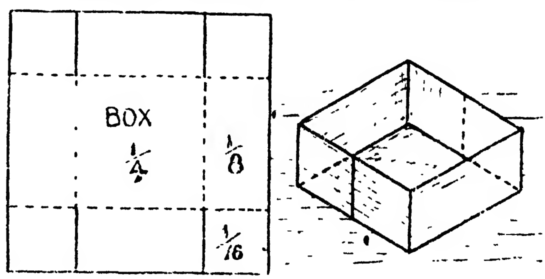


Fig. 13

Box (Fig. 13). Use a 4 in. square of paper. Rule a line level with each side and 1 in. away from it. Cut the heavy lines to form the flanges, and fold along the dotted ones. Paste the flanges to the turned up side on the outside.

Fireplace (Fig. 14). Use a 4 in. square paper. Divide into four oblong quarters; divide one of the quarters into four narrow strips, each a quarter of an inch wide. Cut these up as indicated, and paste strips to form the fireplace (as in Fig. 14). Only three narrow strips are required for the bars.

Problems and Exercises—I.

1. The line in Fig. 1 contains 8 eighths; how many would a line three times the length contain ?

2. Show by a line that half of a line is the same length as two quarters and four eighths. Prove by marking a strip of paper into halves, quarters, and eighths.

3. How many eighths are there in 3 in., 4 in., 6 in. ? How far is it round the crown ? (Fig. 12).

4. Draw a line 6 in. long and show halves, quarters, and eighths. Cut a strip the same length and make it into a measure showing these parts.

5. Cut out of a 6 in. square strips equal to $\frac{1}{8}$ and $\frac{3}{8}$. Show that the remainder is half of the original square and equal to $\frac{4}{8}$.

6. Use some of the strips of Fig. 5 to make a frame that is half the size of the original square. Find how far round the inside it is.

7. Make a square with two of the triangles of Fig. 6, and show that this is half the original square. Do the same with four of those in Fig. 7. Show that these two squares are the same size, and hence that two quarters are equal to four eighths.

8. In Fig. 10 how much of the whole square is the tower, the door, and the body of the church ? How long is each side of the tower ? How far round the whole figure ?

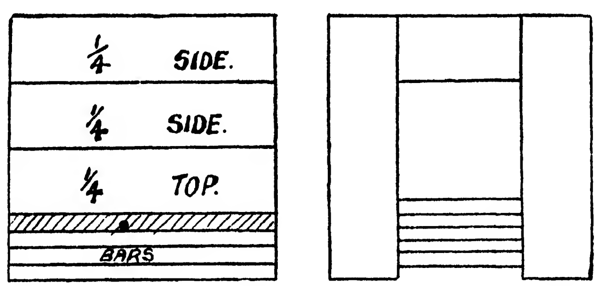


FIG. 14

9. How much of the paper is waste in Fig. 11 ? How much is used for one leg and for two ? For the back and the seat, separately and together ?

10. Show that the waste in Fig. 12 is a quarter of the whole oblong by resolving it into an oblong. Show that the three points are equal to the waste pieces.

11. In the box (Fig. 13) find how much of the whole square is actually used in construction and how much is seen in the box itself.

12. What proportion of paper is used for the bars in Fig. 14 ? For the sides and the top ? How much of the top is seen and how much hidden ?

3. AREA FOLDING OR CUTTING.

The following exercises (Fig. 15) may now be given to prepare the way for the lessons on the square inch by allowing children to deal with areas without knowing they are doing so. The word

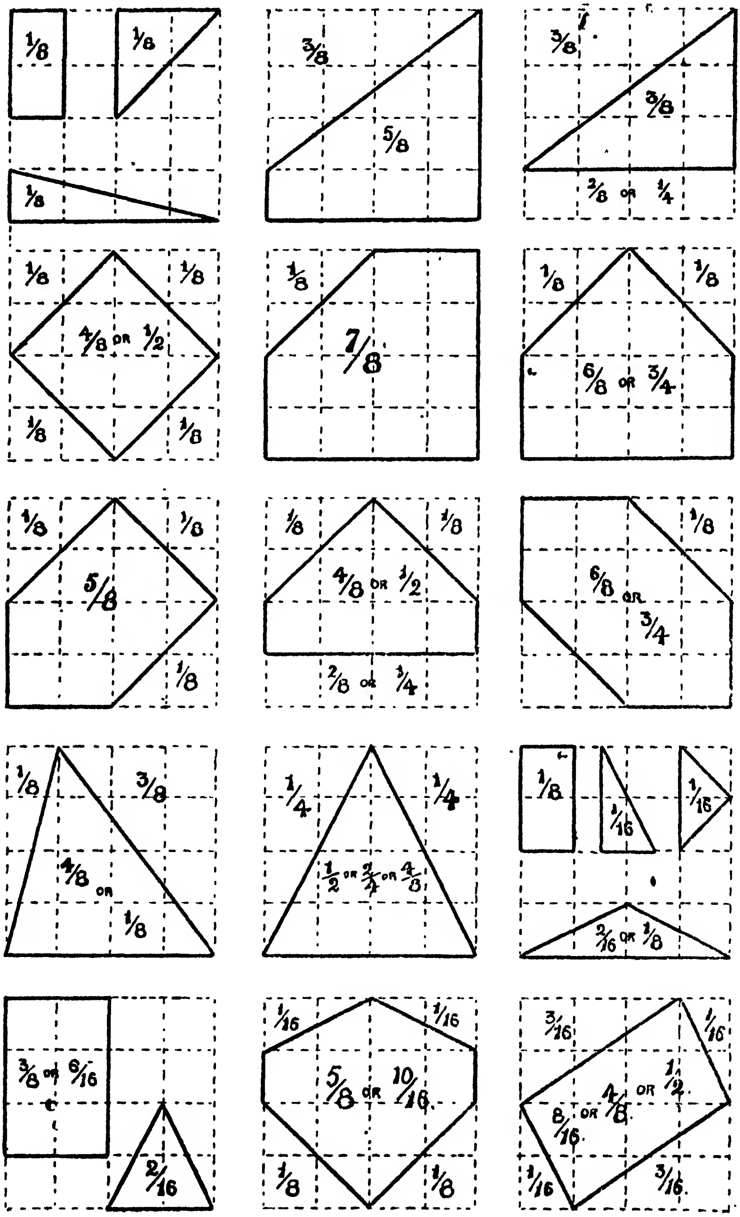


Fig. 15

"area" may not be mentioned. Use 4 in. squares of paper, and take this as the unit.

Innumerable questions and problems may be set on the above figures. Some of the examples may at first sight appear to be too difficult for young children, but if the squares are ruled up into square sixteenths, as shown in diagram, they will become very fascinating problems and the difficulties will largely disappear. It should be always kept in mind that

to make the quarters	the halves	were halved;
,, ,, ,, eighths	,, quarters	,, ,, ;
,, ,, ,, sixteenths	,, eighths	,, ,, ;

and in cases of difficulty the reasoning should commence with the unit and proceed down to the part, *e.g.*, take the last square in Fig. 15: it is required to fold a four-sided figure whose corners are not square, but whose size is eight-sixteenths of the whole square. It will be seen that the $\frac{1}{16}$ portion is half of an oblong eighth, and as $\frac{1}{8}$ is the same as $\frac{2}{16}$, then half of this is obviously $\frac{1}{16}$. The same reasoning applies to the $\frac{3}{16}$ portion. Add the portions cut away—$\frac{3}{16} + \frac{1}{16} + \frac{3}{16} + \frac{1}{16} = \frac{8}{16}$. Now cut off the corners and superpose on the figure formed, and it will be found that the several parts together are equal to the figure formed. Then the $\frac{8}{16} = \frac{1}{2}$.

Many very simple figures leading up to these more difficult ones should be worked first. The problems may be given in something like the following form.

Problems and Exercises—II.

(*Note.*—In all these exercises, rule each square into sixteen small squares.)

1. From the unit square fold as many eighths of different shape as you can.

2. From these make sixteenths.

3. Make one fold and let there remain $\frac{5}{8}$. How much is folded away? How do you know it is $\frac{3}{8}$?

4. Show that $\frac{1}{2}$ is the same as $\frac{2}{4}$, and $\frac{2}{4}$ the same as $\frac{4}{8}$, and $\frac{4}{8}$ the same as $\frac{8}{16}$.

5. Fold away two triangular pieces each equal to $\frac{1}{8}$. How much is left? Fold away three; how much left?

6. Fold a five-sided figure, which shall form a half of the square. (There are three folds, two an eighth each and one a quarter.)

7. Make two different triangles (two folds to each), each one being half the square.

8. Cut out an oblong which is $\frac{3}{8}$ of whole. From this cut a triangle which is $\frac{3}{16}$ of whole.

It will be seen that there may be various correct answers to each question, and the children should be given credit for any answer so long as it fulfils the conditions laid down.

4. THE SQUARE INCH.

Supply each child with a 4 in. square of paper. Cut off an oblong quarter, as in Fig. 16, and divide each long side into four equal parts. Join the opposite points and cut along the drawn lines.

Examine one of the squares and prove it to be such :—

(*a*) Strips of paper } for sides.
(*b*) Ruler }
(*c*) Folding (for sides and corners).

Examine each one similarly ; superpose, etc. ; they are all squares, and all same size.

Let them cut another strip off ; superpose the four squares on this. Observe that there is the equivalent of four small squares in the strip.

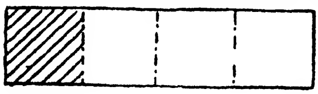


FIG. 16

Make oblongs of two strips, three strips, and a square of four strips. Observe in each case how many of the small squares it takes to make the figure—four to a strip, and one, two, or three strips.

Examine the small square again more closely, and write down what has been found :—

(*a*) Each square is same size and shape.
i.e. { (*b*) Each is a square.
(*c*) Each side is 1 in.
Hence name—one "SQUARE INCH."

Then use a folded 4 in. square. How many square inches in this ? Put two, three, and four small squares together to form oblongs. Ascertain the number of square inches in each. Build up four squares and nine squares together and form squares.

Make oblongs of 2, 4, 6, 8, 12, 15, and 16 sq. in. In each case insist on children giving :—

(*a*) Length of sides in inches of the squares or oblongs formed, and

(*b*) The number of square inches used to build up the square or oblong.

Have ruled and cut out various squares, oblongs, and triangles of a given number of square inches. Let the children test these with one or more of the original square inches already tested.

Gradually lead children to see how many square inches of paper there are in various squares and oblongs submitted to them, *e.g.* :—

Red.	*Blue.*	*Green.*	*Yellow.*	*Brown.*
2 in. × 3 in.	4 in. × 5 in.	2 in. × 4 in.	3 in. × 5 in.	5 in. × 5 in.
3 in. × 4 in.	5 in. × 6 in.	4 in. × 6 in.	3 in. × 6 in.	6 in. × 6 in.

When sufficient examples have been studied, the teacher should

direct the children's attention to their multiplication tables, and suggest a quicker method of discovering the number of square inches in a figure. Teacher should further question as to the numbers that have to be multiplied together in order to make 2, 4, 6, 8, 12, 15, 16, 20, 24, etc.

Observe the relation between these numbers and the "areas" of the various figures examined.

Let children rule up the squares and oblongs into square inches by dividing each side into inches and joining the opposite points. Then, using the known square inches as templates, find how many are required to cover a certain "area."

Explain that *area* is the word used to denote the surface or amount of space covered by a figure.

Again give simple figures for the children to find out how many square inches they contain or to find their *area.*

Following on squares and oblongs, simple triangles should be given. They have already discovered that a triangle is half of an oblong whose base and height are the same, from dealing with the exercises in Area Folding and Cutting.

For further exercise on this, a study of Fig. 15 will reveal a sufficiency of examples for the teacher to invent more for himself.

5. HANDWORK EXERCISES.

Illustrative of the Square Inch.

Chair (Fig. 17). Use an oblong 3 in. × 2 in. of stiff paper. Rule up as in diagram, cut out the shaded portions, and fold as indicated.

Note that for the legs there are crest creases shown, while for the back a furrow crease is indicated.

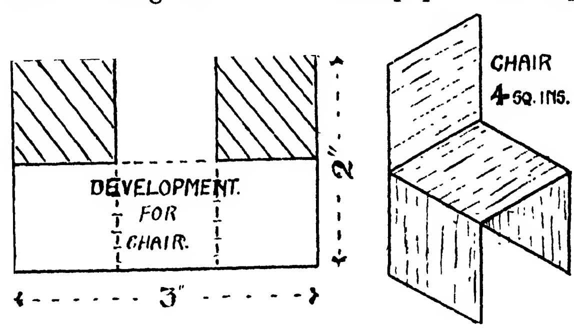


FIG. 17

Tray (Fig. 18). Use a 6 in. square of paper. Draw a line level with each side and 1 in. away. Cut the heavy lines as shown. Fold up the sides and flanges (marked "*a*" in figure), and paste the latter to outside of sides.

Draught Board (Fig. 19). Cut out 32 red and 32 blue 1-in. squares. Mount these alternately on a piece of cardboard 9 in. square, leaving a border of half an inch all round. Let children observe that the "area" of the board itself is 64 square inches, *i.e.*, 8 in. × 8 in.

8 columns with 8 in each column = 64 sq. ins.
or, 8 rows ,, 8 ,, row = 64 ,, ,,

The area of the cardboard is then 9 in. × 9 in. = 81 sq. ins. Find the area of the border practically and verify by calculation. Make up into strips as in diagram (Fig. 20), one 9 in. long and 1 in. wide, the other 8 in. long and 1 in. wide.

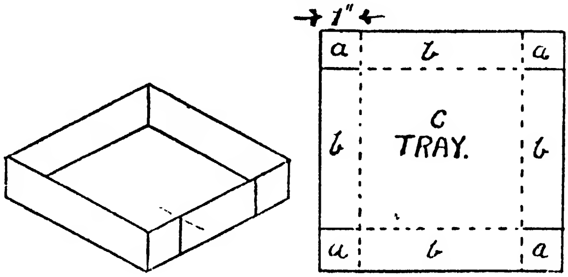


FIG. 18

$$\left.\begin{array}{l}(a)\ 9 \text{ in.} \times 1 \text{ in.} = 9 \text{ sq. in.}\\ (b)\ 8 \text{ in.} \times 1 \text{ in.} = 8 \text{ ,,}\end{array}\right\}\text{Total 17 sq. ins. border.}$$

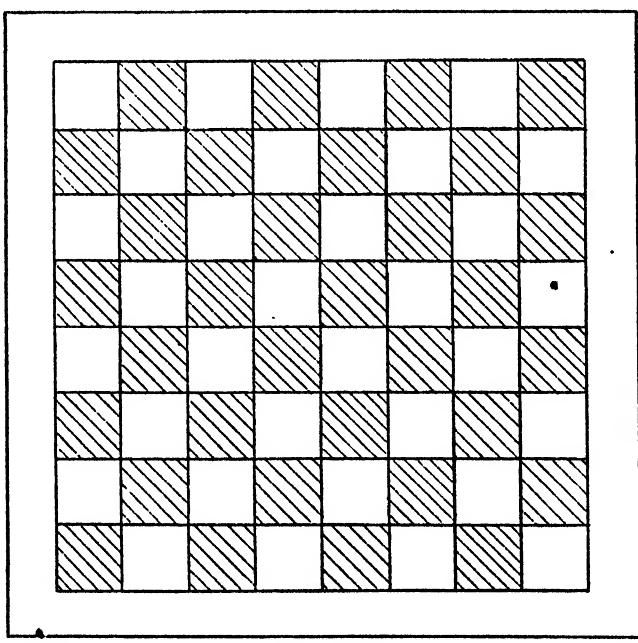


FIG. 19

Then total area of board and border is :—

64 sq. ins. + 17 sq. ins. = 81 sq. ins

Problems and Exercises—III.

1. How many square inches are there in a piece of paper 5 in. long and 3 in. wide ?

2. If a piece of card contains 12 sq. in. and it is 4 in. long, how wide is it ? If 2 in. wide, how long is it ?

3. One oblong is 3 in. long and 4 in. wide, another is 5 in. long and 2 in. wide. Find the area of each, see which is the bigger, and by how much.

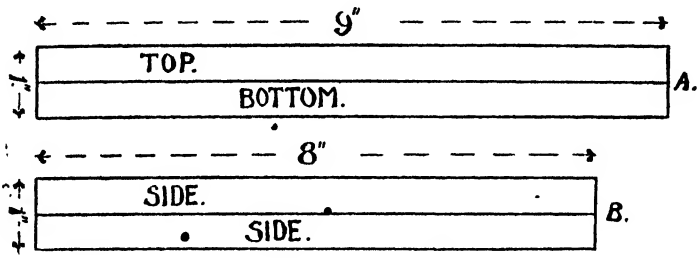


FIG. 20

4. An oblong has an area of 8 sq. in.; find the lengths of its sides (two answers).

5. If one side of an oblong is 5 in. long and its area is 20 sq. in., what is the length of the other side ?

6. A square has an area of 25 sq. in.; how long is its side ? How far round is it ? Make a triangle half the area.

7. If a penny stamp is 1 sq. in. in area, what area can I cover with a shillingsworth of them ? With nine pennyworth of halfpenny ones ?

8. How many stamps should I require to cover a post-card completely, if it were 5 in. long and 3 in. wide ? How far round the post-card would it be ?

9. How many square inches of paper are used to construct the chair (Fig. 17) ? How many waste ? How many in the oblong at first ?

10. What is the area of the bottom of the tray in Fig. 18 ? The four sides ? The four flanges ? What is the difference in the areas of the original square and of the bottom ?

11. Make a triangle which is one quarter the area of the draught-board. Prove it by drawing the reconstruction.

12. What is the difference in area of the development for the chair and the bottom of the tray ? And the difference between the square used for the tray and the draught-board ?

13. How far is it round the chair (Fig. 17) when standing up ? How high is it ? How wide ?

14. How far is it round the tray (Fig. 18) ? Round the square ?

Round the draught-board? How many trays will just cover the 64 squares?

15. How much of the draught-board is the tray? And how many times the area of the tray is the draught-board?

16. Find how many chairs like Fig. 17 could be made from a square as large as the tray. What would be the area of the waste paper?

17. A fourpenny tin of paint will cover 24 boards, each board being 5 in. long and 4 in. wide. What area is painted? How much does it cost to paint 240 sq. ins.? What areas can I get painted for a penny, and for threepence?

6. HALF A SQUARE INCH.

(1) The Oblong Half.

Tell children to make two or three "square" inches as before. Take one and divide it into two oblong halves (Fig. 21), measuring and ruling before cutting. Refer to lesson on halves, etc. Note

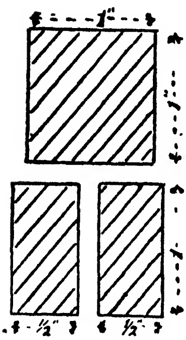


FIG. 21

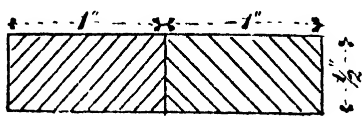


FIG. 22

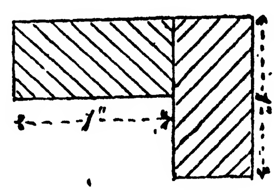


FIG. 23

that the oblongs now cut are each half of the original square. Since this square was a square inch, then each of the oblongs must be half a square inch.

Place the two halves end to end (Fig. 22), and obtain an oblong which is still the same area—occupies the same amount of surface. Place in various other positions (*e.g.*, Fig. 23), and obtain other figures, coming to the same conclusion each time, viz., that as long as it covers the same room (space) as 1 sq. in. does, it is the same as 1 sq. in. *in area*.

Select simple oblongs, as in Fig. 24, incorporating the oblong ½ in. Have them built up with known square inches and half inches, and the area found practically. Also have squares and oblongs drawn according to simple dimensions, ruled up into square inches, and the area found (Fig. 25).

Reconstruct the half square inches into whole ones, and add to the undivided square inches.

(2) **The Triangular Half.**

Let children now divide a square inch into halves another way

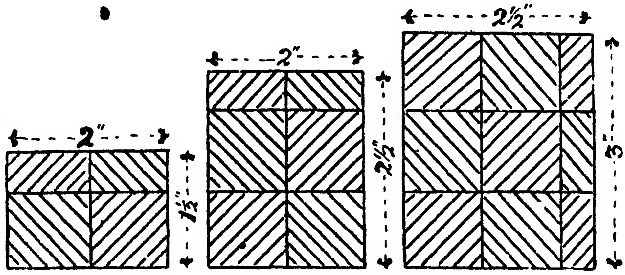


Fig. 24

—refer to previous lesson on half a square—that is, diagonally (Fig. 26). Again we have half a square inch, but this time a "three-cornered figure."

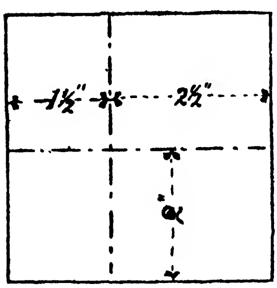


Fig. 25

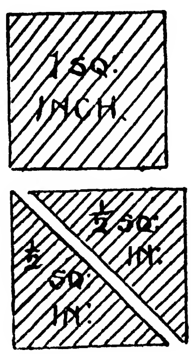


Fig. 26

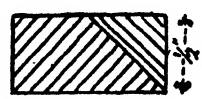


Fig. 27

Cut out and reconstruct the triangular half into an oblong half (Fig. 27), and compare, by superposing, with the former (oblong) half square inch (Fig. 21), and *vice versa*.

Cut a 2 in. square diagonally (Fig. 28). Let the children estimate the area of the triangular half. Test it with the square inch and the triangular half inches—lay them on top of this larger triangle and state the area. Treat a 4 in. square in the same way, and let children make a whole square inch with the two triangular half inches in Fig. 28, and note that there are now two *square* square inches (forming an oblong, which is equal to the oblong half of the corresponding square). Deal with the four triangular

half inches of the 4 in. squares in the same way. Compare the areas of the triangles with the corresponding squares.

Take an oblong 4 in. × 6 in. and rule it up into square inches. Cut it diagonally and shade one of the triangles (as in Fig. 29). Cut up the shaded triangle into its various pieces, and construct the new oblong with them. Notice that each of the shaded portions helps to complete an exact number of square inches.

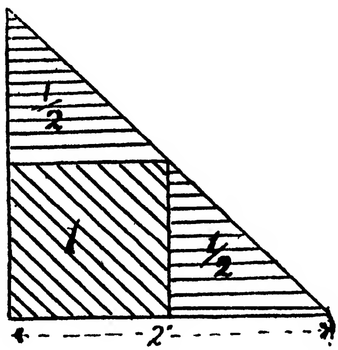


FIG. 28

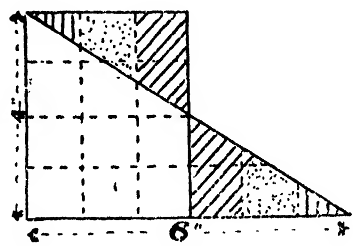


FIG. 29

Hence show that the whole triangle is equal to the new oblong, and this being half of the original oblong, the latter and the former are, therefore, equal in area.

Many of these examples would be advantageously dealt with on squared paper (1 in. squares), either as alternative or supplementary exercises.

7. HANDWORK EXERCISES.

Illustrative of Half a Square Inch.

Barn (Fig. 30). Paper 4 in. × 2 in. cut from a 4 in. square. Rule up into square inches; rule off the corners and cut away shaded triangles. With these form a square. Note its area and the area of the remainder.

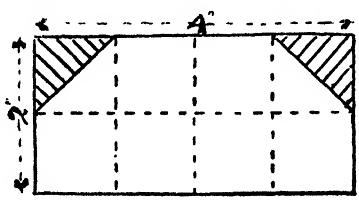


FIG. 30

House (Fig. 31). Use paper 5 in. × 4 in. Rule up the paper into square inches, as in diagram, and cut away the two large triangles to give the shape to the roof. Form a square with these triangles, and note its area. Paste house-end and the square formed of the waste pieces in exercise book with other practical work.

Church (Fig. 32). Build up the church with pieces cut from previous examples. Find the area of each piece separately, and

make note of each. Paste in book and have the statement underneath, so :—

$$\begin{aligned} \text{Area of spire } a &= \tfrac{1}{2} \text{ sq. in.} \\ \text{,, \quad ,, } b &= \tfrac{1}{2} \text{ \quad ,,} \\ \text{,, \quad body } c &= 4 \text{ sq. ins.} \\ \text{,, \quad ,, } d &= 3\tfrac{1}{2} \text{ \quad ,,} \\ \hline \text{Total area} &= \underline{\underline{8\tfrac{1}{2}}} \text{ \quad ,,} \end{aligned}$$

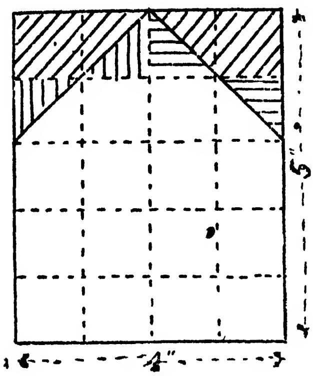


FIG. 31

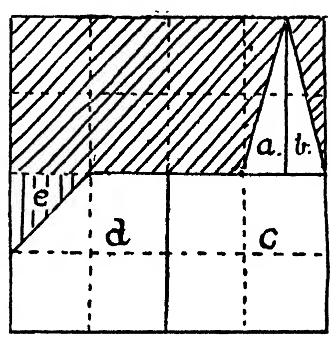


FIG. 32

Resolve the spire into an oblong square inch and again into a *square* square inch. Fill up "*e*" with one half of the spire, thus showing an oblong of 8 sq. ins. area and $\frac{1}{2}$ an inch left, making a total of $8\frac{1}{2}$ sq. in.

Basket (Fig. 33). Use a 6-in. square of paper. Divide each side into four equal parts, and rule as shown in the diagram. Cut out the waste material (shaded) and along the heavy lines shown. Fold up sides and ends as necessary, also the flanges (*f*); paste the latter outside the ends. This forms the body of the basket. The handle, as shown, should be folded so as to be half the width, and pasted down so as to be double in thickness. Fix handle on to the inside of basket right down to the bottom.

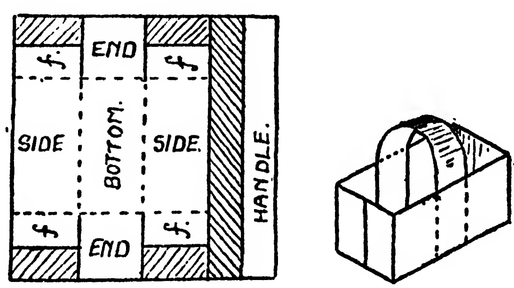


FIG. 33

8. MULTIPLICATION BY TWO FIGURES.

In dealing with multiplication with two figures in the multiplier, it is suggested that the following plan be adopted for simplicity and the logical reasoning of the process.

Refer to "areas" previously worked, and suggest that larger numbers will be used than have been the case so far. Suppose, for example, we wish to find the area of the door in square inches. We measure it and find it is 48 in. wide and 94 in. high.

Analysing the multiplier, we find that we have to multiply by 40 and then by 8. Multiply by 40, first placing a "0" under the units and multiplying by 4. Indicate at the side that this is forty times. Next multiply by the 8 in the usual way. Add the two lines, and indicate that the result is forty-eight times the number.

Th	H	T	U	
		9	4	
		4	8	
3	7	6	0	= 40 times
	7	5	2	= 8 ,,
4	5	1	2	= 48 ,,

In order to prove this answer correct, let the sum be worked in two parts. First multiply 94 by 40, then again by 8. Add these two answers, and notice the similarity in the working to the example worked as one sum.

$$\begin{array}{r} 94 \\ 40 \\ \hline 3{,}760 \end{array} = 40 \text{ times } 94 \qquad \begin{array}{r} 94 \\ 8 \\ \hline 752 \end{array} = 8 \text{ times } 94$$

$$\begin{array}{rl} 3{,}760 & = 40 \text{ times } 94 \\ 752 & = 8 \quad ,, \quad ,, \\ \hline 4{,}512 & = 48 \quad ,, \quad ,, \end{array}$$

Problems and Exercises—IV.

1. Take a 4-in. square of paper, rule up into square inches, as shown in Fig. 34 A. Cut as many different shapes and different areas as possible. Draw these in your exercise book, and note their area and how you found the area.

2. Show that B, C, and D are all equal in area; also that E, F, and G are equal, and half the area of B, C, and D. From B, C, D, E, F, and G together, how many 3-in. squares could you form? How many oblongs 3 in. × 2 in.? Draw diagrams to prove this.

3. If the price of A is 1s. 4d., how much are B, C, D, E, F, and G worth ?

4. I pay a shilling a square inch for silver, how much ought I to get for half a crown ? For half a sovereign ? A florin, etc. ?

5. If B, etc., is paper sufficient to cover a wall, how much wall can I cover with A, E, F, etc. ?

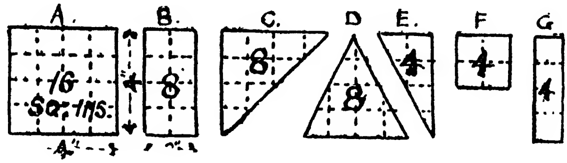


FIG. 34

6. Suppose F is a tile for a floor, how many shall I require to cover a floor three times the width and twice the length of B ? What would be the area of that floor ?

7. What fraction of the oblong in Fig. 30 is waste paper ? What fraction forms the barn ? What is the area of the waste and the barn ? Resolve the barn into seven whole square inches.

8. Find the distance (to an eighth of an inch) round the barn and the house. What is the area of the triangular portion of the latter ?

9. Find how far it is round the church, Fig. 32 (to a sixteenth). How many times is the area of *c* greater than that of *a*, *b*, and *d* ?

10. In Fig. 33 (Basket), what is the area of each end ? Each side ? The bottom ? The flanges ? The waste material ? Total area of paper used in the basket ? Prove by subtracting the waste from the whole square.

11. A 6 in. square has a border ½ an inch wide all round it. What is the area of the border ? Prove it by drawing or cutting out.

12. A book 7½ in. long and 5 in. wide has a label 2½ in. × 3 in. on the front of it. Find the area of each, and also find how much of the book cover can be seen.

13. The sides of a square are 1 in. long ; suppose each stands for 1 yd. Draw an oblong which will represent one 9 yds. long and 4½ yds. wide.

14. A sheet of writing paper is 6½ in. long and 4 in. wide. Find the area of a penny packet of this paper if there are 12 sheets to the packet ? (Answer in square inches.)

15. There are 25 envelopes in a packet ; if they are 4 in. long and 3 in. wide. what is the area of the whole lot ? (Answer in square inches.)

16. A board measures 12 ft. long and 2½ ft. wide. Cut out a paper to represent it, taking 1 in. for every foot ; find its area. Also work out in figures.

17. A door is 3½ ft. wide and 7 ft. high. Represent this in

paper when 1 in. stands for each foot. Find the area of the door.

18. A door has four panels; each of the top ones is 28 in. long and 12 in. wide, how many square inches does it contain? Each of the bottom panels is 12 in. wide and 18 in. long; find its area and the total of all four.

19. A pane of glass is 9 in. wide and 13 in. long; find its area. If there are 24 such panes in a school window, what is the area of the glass in square inches?

20. In a book there are 72 pages. If each page contains 24 square inches of printing, what is the area of the printing in the whole of the book?

21. If the glass mentioned in Question 19 cost 2d. a pane, how much did the window cost?

22. A school desk is 11 in. wide and 42 in. long. Find the area in square inches.

23. Assuming that one quarter of the earth's surface is land and three-quarters water, fold a square of paper in such a way as to show this. Write on each of the four parts what it represents.

9. ANGLES AND TRIANGLES.

(1) Angles.

Angles have previously been spoken of as "corners" and "square corners." The time has now come to use the more correct term —angle—instead of corner.

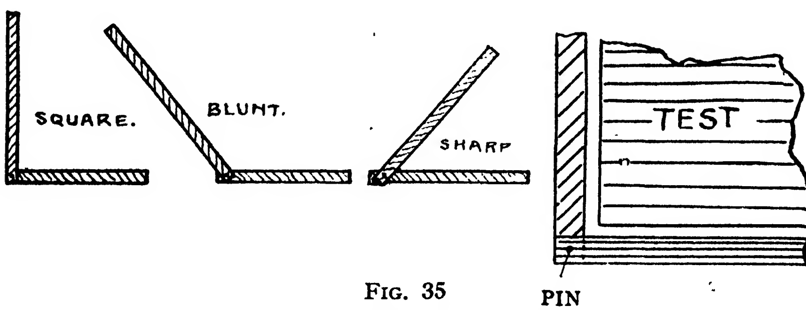


Fig. 35

Cut two strips of carton paper ½ an inch wide and 6 in. long. With these form a "square" angle or *right* angle (Fig. 35). Place a pin through the two papers into the desk and move strips about this pin. Here explain that *right* means a conformity to truth; straight, just, and true. Compare with walls, windows, books, etc. Notice in the case of books, papers, etc., that the angle still remains "right," though it may be placed in any position.

Tell children to turn the strips about so that the point of the angle turns upward, downward, to right and left, etc. Test each time, and correct, if not exact, with a square previously tested, as in Fig. 35.

Let children understand that a right angle is the standard from which others are reckoned. Some are smaller, others larger than the standard, and are named accordingly. Use the terms "sharp" and "blunt" for acute and obtuse respectively; at any rate for the present (Fig. 35).

Tell children to examine some of the triangles obtained in the earlier exercises, and place their strips so as to make angles the same size as the various angles in these triangles. Test with the triangle copied, and alter if necessary.

The teacher should use a pair of blackboard compasses to demonstrate with, and let the children copy as near as possible the angle he makes with the arms of the compasses. Compare these angles with the standard or right angle, and notice which are less and which are greater.

Those angles less than the standard are sharp angles and those greater are blunt. Compare a pin point, spiked railings, and a spear head; ascertain why sharp. Refer also to a blunt knife. It is dull, not keen or sharp—blunt. Hence the names.

The term angle may be regarded as the surname, while blunt and sharp are the Christian names.

Let children point out examples of the various angles in the room, school, and in familiar objects. Elicit *why* blunt or sharp; whether very blunt or very sharp, etc.

Have paper cut into various angles and pasted in exercise books, with their names written on or underneath them.

(2) Triangles.

Consider the words tripod and tricycle. Write on blackboard; sketch or show the objects, and note a common property of each. Explain that tri- means three, and so "tripod" means three-footed and "tricycle" having three wheels. Also "triangle" means three angles or three corners.

Now let children draw a triangle and cut it out; examine it and find out all they can about it. It has three sides and three angles. Examine the angles; find what kind they are; whether right, blunt, or sharp. Test them with a right angle by superposition, and ascertain—

If the right angle exactly fits . . . . it is a *right* angle;
,, ,, ,, covers it . . . . . . ,, ,, *sharp* ,, ;
,, ,, ,, does not cover it . ,, ,, *blunt* ,, .

Try the reverse of this, and verify the facts ascertained previously.

Tell children that there are several kinds of triangles named after the angles which help to form them. (Omit at this stage the triangles named from their sides.) Tell them to draw and cut out a triangle having in it a right angle (Fig. 36 A). This triangle is named after this angle a ***right-angled triangle.*** Compare with surname and Christian name, as was done with the angles.

Now cut out a triangle containing a blunt (or obtuse) angle (Fig. 36 B); this is an *obtuse-angled triangle.*

Examine and compare these two triangles, and notice that each

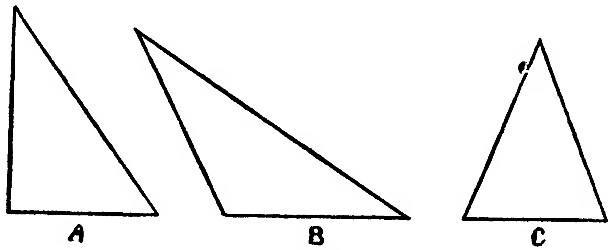


FIG. 36

contains two sharp angles. Have a triangle cut now that has *all* its angles sharp, or less than a right angle (Fig. 36 C); this is a sharp or *acute-angled triangle.*

Have an example of each pasted in the exercise books and named.

Problems and Exercises—V.

1. Make a right angle with strips and with another strip divide this into two acute angles: (*a*) both the same size, (*b*) one twice the size of the other.

2. Draw an obtuse angle which shall be equal to two acute angles.

3. Make an acute angle with your strips, and an obtuse angle which is twice the size.

4. Say why it is necessary to have a knife edge acute; a spear point sharp and not blunt; and a nail with a point.

5. Find the distance round the right-angled triangle, the obtuse-angled triangle, and the acute-angled triangle (Fig. 36).

6. Find the longest side of three triangles; also the greatest angle. Where does the longest side always appear?

7. Can you have a triangle with two right angles in it? Try to make one. Why cannot you do it?

8. Make a right angle with lines $1\frac{1}{2}$ in. and 2 in. long; join these two ends, and make it into a triangle. How long is the longest side? How far is it round?

9. If this triangle (Question 8) is half an oblong, make the oblong. What is its area? Its distance round? Cut the triangle out and prove it.

10. If the arms of an obtuse angle in a triangle are 2 in. long, find the middle of the longest side and join this with opposite angle. Cut along this line and make an acute-angled triangle.

11. From a 4 in. square cut a right-angled triangle, an obtuse-angled triangle, and two acute-angled triangles. Find the distance round each and the area of the first.

12. Cut out any oblong equal in area to a 4 in. square. Are they both the same distance round ? If not, which is the greater, and by how much ?

10. THE FOOT.

Having now dealt with inches fairly fully, it will be well for the children to proceed with the name given to these when 12 of them happen to come together in linear form — the FOOT.

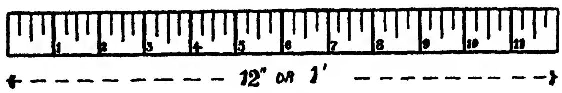


FIG. 37

Let children cut strips of paper 1 in. wide, say 7 in. long ; paste two strips together to form one about twice as long, allowing an overlap of half an inch (Fig. 37) ; commence and mark off from the ruler in inches up to twelve. Cut off exactly 12 in. long, and number the inches up to 11 —it is obvious there is no room to put the number 12 in—remembering that the *space* between one line and the next is the inch. This strip is now 1 foot long.

Next mark it off into half inches and quarter inches. One inch may be marked off into eighths.

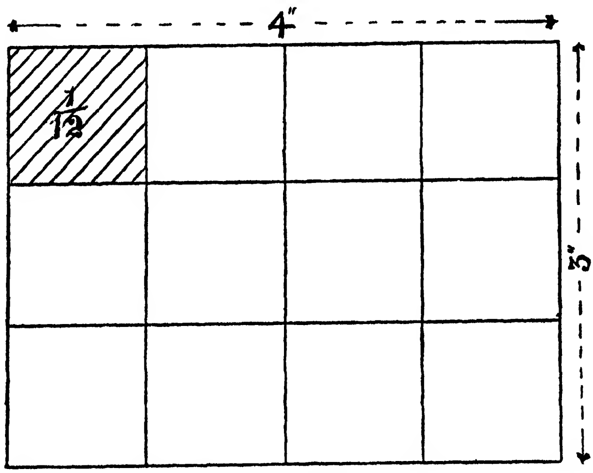


FIG. 38

Give exercises in measuring objects over 1 foot in length, *e.g.*, length of desk, height of door, length and width of the blackboard, easel, etc.

When a strip was divided into :—

(*a*) 4 equal parts, each part was named a *fourth ;*
(*b*) 8 „ „ „ „ „ „ an *eighth ;*
(*c*) 16 „ „ „ „ „ „ a *sixteenth.*

When a strip like Fig. 37 is divided into 12 equal parts, each part is called a TWELFTH. Have an oblong 3 in. × 4 in. cut from a 4 in. square and ruled up as in Fig. 38. Find the area, perimeter

etc., and show that as there are 12 sq. in. in the paper, each one must be a twelfth—a twelfth of the area of the paper.

11. SIMPLE SCALE.

Following on the principle of letting certain lengths and areas of paper represent certain quantities, *e.g.*, money, the representation of one length by another naturally follows.

Have a 1 in. strip, ½ an inch wide, cut off separately, and let this represent 1 foot. Draw a line 1 in. long and let this represent 1 foot.

Give various exercises on this same representation to show 6 in., 9 in., 1 ft. 6 in., 2 ft., 3 ft., 6 ft., up to 12 ft.

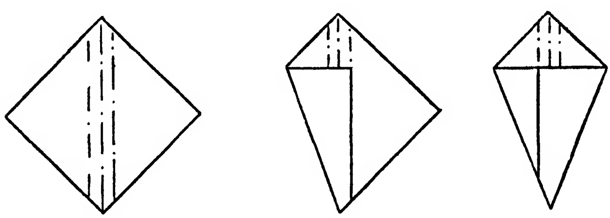
Represent in strip and line heights or lengths of boys, desks, doors, pencils, etc., 1 in. to represent 1 foot.

Proceed next with half scale and quarter scale. (Teacher to exclude awkward examples.)

Length of schoolroom, street, etc., any object with which children are familiar and which is within their capabilities should be afterwards represented in simple scale.

12. HANDWORK EXERCISES.

Sweet Cup (Fig. 39). Use a 6 in. square of paper for this model. Draw one diagonal and a line on either side of this ⅛ in. away from it (*a*). Fold up the left corner of (*a*) to fall along line shown in (*b*). Do the same with other corner. This will form an overlap of ¼ in. (*c*); paste this down and fold the little piece at the bottom round the back, and paste it down. Care should be taken in pasting, or the cup will stick on the inside.



A B C

FIG. 39

Parachute (Fig. 40). Fold an 8-in. square diagonally and diametrically. Sketch in the curves and cut out the waste material. From this waste cut out eight squares, each of 1 in. side, to fasten on the cotton to the paper. Cut each piece of cotton a foot long, and tie all in a knot at the loose end.

Problems and Exercises—VI.

1. How many inches in a foot? In half, quarter? In 2 ft., 3, 5, 6?

2. How many half inches in a foot? In 3, 4? In half a foot, quarter?

3. In 36 inches, how many feet ? In 60, 48, 24 ? In 30 in., how many whole feet, and how many inches left ? In 19, 27, 45 ?

4. If I gave 3 in. of ribbon to each of eight boys, how much should I require ?

5. A string is 36 in. long, how many pieces 4 in. long can I cut from it ?

6. If I buy silk at a penny an inch, how much can I get for 1s., 2s. 6d., 5s. ?

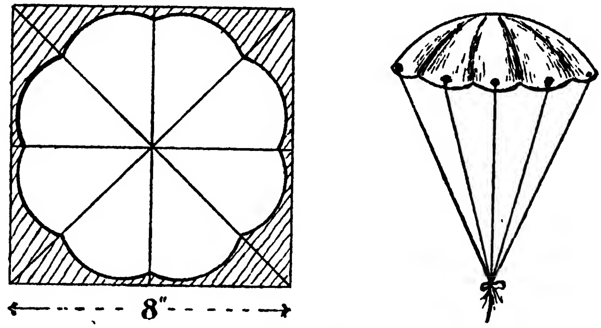


FIG. 40

7. I pay 2s. a foot for Scotch tweed, how much will a suit cost me if there are 18 ft. in it ?

8. Cut out a piece of paper to represent a brick ; children to measure length and width, and form an oblong half the width, but same area.

9. A matchbox case is $2\frac{1}{2}$ in. long, $1\frac{1}{2}$ in. wide, and $\frac{3}{4}$ in. thick. Cut out a piece of paper large enough to make it, using some for a fastener.

10. Cut out a piece of paper to represent your drawing book on quarter scale. (That is quarter the length and quarter the width.)

11. Measure the distance round a brick (three ways) in inches. Find how many feet and remaining inches each is equivalent to.

12. Measure the length and the width of the room. From this find the distance round.

13. SIMPLE FRACTIONS.

Addition and Subtraction—*half, quarter, and eighth.*

(*a*) $\frac{1}{2} + \frac{1}{4} + \frac{1}{8}$; (*b*) $1 - \frac{1}{8}$.

Rule or fold up an oblong piece of paper into square eighths (Fig. 41). Mark out $\frac{1}{2}$, $\frac{1}{4}$, and $\frac{1}{8}$. Ascertain the number of eighths in each of these, and write down on the paper (Fig. 41).

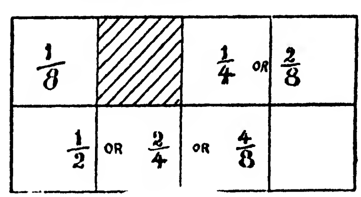


FIG. 41

Set down in usual form—

$$\frac{1}{2} + \frac{1}{4} + \frac{1}{8} = \frac{4}{8} + \frac{2}{8} + \frac{1}{8}$$
$$= \frac{7}{8}.$$

Shade in or cut away the eighth not required in the sum. Reverse the question, and from the unit oblong take away $\frac{1}{8}$; 7

eighths ($\frac{7}{8}$) remain. In this way, addition and subtraction can be taught concurrently. Set down as follows :—

$$1 - \tfrac{1}{8} = \tfrac{8}{8} - \tfrac{1}{8}$$
$$= \tfrac{7}{8}.$$

(*c*) $1\frac{3}{4} + \frac{5}{8} + 2\frac{1}{2}$.

Cut and fold six 4-in. squares as in Fig. 42. Resolve into their respective eighths and count the number of whole units and eighths (3 units and 15 eighths). This gives 3 units and $1\frac{7}{8}$ units, or a total of $4\frac{7}{8}$.

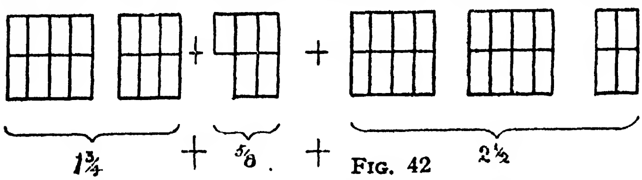


FIG. 42

Set down and work in proper form :—

$$1\tfrac{3}{4} + \tfrac{5}{8} + 2\tfrac{1}{2} = 3 + \tfrac{3}{4} + \tfrac{5}{8} + \tfrac{1}{2}$$
$$= 3 + \tfrac{6}{8} + \tfrac{5}{8} + \tfrac{4}{8}$$
$$= 3 + \tfrac{15}{8}$$
$$= 3 + 1\tfrac{7}{8}$$
$$= 4\tfrac{7}{8}.$$

(*d*) $2\frac{3}{8} - \frac{3}{4}$.

Fold and cut $2\frac{3}{8}$ and $\frac{3}{4}$ as shown in Fig. 43. Resolve into eighths as before and note the number in each.

It will be seen that one of the whole units will have to be used in order to subtract the $\frac{3}{4}$. The latter contains 6 eighths; and the odd $\frac{3}{8}$, and $\frac{8}{8}$ of a unit leaves one whole unit and $\frac{5}{8}$ of another. Set out in sum form :—

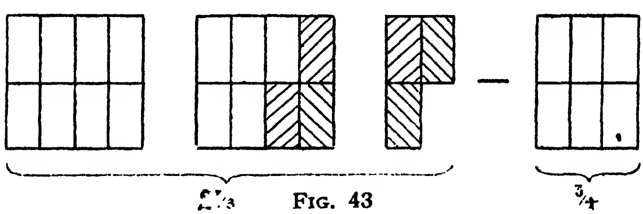


FIG. 43

$$2\tfrac{3}{8} - \tfrac{3}{4} = 1 + 1\tfrac{3}{8} - \tfrac{3}{4}$$
$$= 1 + \tfrac{11}{8} - \tfrac{6}{8}$$
$$= 1 + \tfrac{5}{8}$$
$$= 1\tfrac{5}{8}.$$

Use strips and squares alternately or conjointly, and verify with drawn lines. Give various exercises and problems on the lines of Exercise VIII, Part 1, but including the eighth.

Reverse the questions whenever thought desirable, so establishing the connection between addition and subtraction. Many questions can also be dealt with in the previous Handwork Exercises.

PART III.

SYNOPSIS.

1. SIMPLE DECIMALS.

Tell children to draw a line 1 in. long in their books and name it the *unit* line (Fig. 1); have other lines drawn 2 in., 5 in., and 10 in. long, and ascertain the number of times the unit line each one is. Question as to the number of times the unit is contained in each, and name, as in Fig. 1.

Refer to the names given to other fractions, *e.g.*, fourths, eighths, sixteenths, twelfths, etc., and why so named. Ask for the name of fraction given to the one, two, and five-inch line when the 10-in. line is taken as the unit (one-tenth, two-tenths, and five-tenths). Do not deal with the fraction in lowest terms at present.

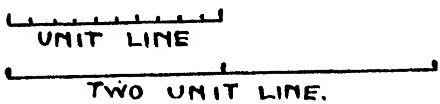


Fig. 1

Tell children to draw now a line 2 in. long, and divide it into twenty equal parts by using the "tenths" marked on their rulers. Let them find these on the ruler for themselves.

Verify, by counting, the parts each inch is divided into—in this case ten. Each part, then, is one-tenth of an inch.

Set down the numbers 10 in., 12 in., and 21 in., and question on the local value of these figures.

10 in. = 1 ten + 0 units.
12 in. = 1 ,, + 2 ,,
21 in. = 2 tens + 1 unit.

Again refer to the rulers and the inch and tenths. Suppose we have a line $2\frac{1}{10}$ in. long. Draw this. How are we to write down the length? Set down so:—

21 = 2 tens + 1 unit.

Now set down $2\frac{1}{10}$ in. in the same manner. The two in this case represents 2 units; and the *one* is one-tenth of a unit.

Number.	Tens.	Units.	Tenths.
21 =	2	1	
$2\frac{1}{10}$=	0	2	1

Fig. 2

Refer again to the local value, and that the *units'* column (Fig. 2) is one-tenth of value of the *tens'* column. In the same manner the 1 (for 1 tenth) is placed on the right of the units' figure.

Lead children to see that the figures on the right of the thick line represent parts of a unit—that is, tenths of a unit. While those on the left of the thick line are whole units.

Repeat this process with strips of paper $\frac{1}{4}$ of an inch wide and taking figures different from those above.

Paste these strips in order in the books under the examples taken with the lines.

(1) Square Inch and its Tenth.

Let the children rule strips of cardboard 1 in. wide; cut off pieces 5, 2, and 1 in. long (Fig. 3). Rule up into square inches and mark each unit square inch separately.

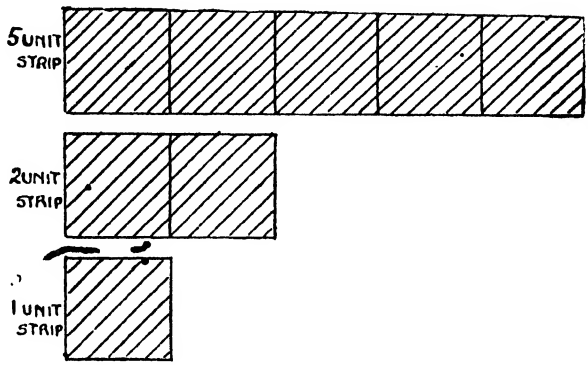


FIG. 3

Supposing the square inch be taken as the unit, then question, as was done when dealing with the line in the previous example.

Take a square inch and divide two opposite sides into tenths; rule across; cut and obtain ten tenth-strips to the unit (Fig. 4).

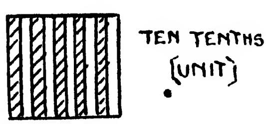


FIG. 4

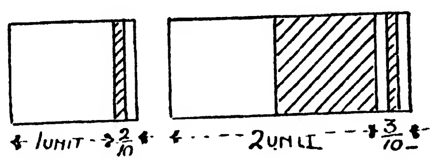


FIG. 5

Tell the children to build up various areas in square inches and decimals of the same, *e.g.*, 1 unit and 2 tenths, 2 units and 3 tenths (Fig. 5). Give plenty of further practice at this stage, increasing in difficulty as necessary. Write down the results as was done previously.

Units.	Tenths.
1	2
2	3

FIG. 6

(2) The Decimal Point.

As there is a difference in value between the figures on the right and left of the heavy line in Fig. 6—revise this point in connection with Fig. 2 here—and as it is not always convenient to make a line between whole units and tenths of units, we place a dot, which is called *the decimal point*, so, 1·2 and 2·3.

Have strips of card cut off any length at random, also lines drawn any length similarly, and measured in inches and decimals to the nearest tenth.

Place several pieces end to end, count units and tenths, and write down the result. Verify by measurement.

Use the same or similar examples as in Figs. 5 and 6, and repeat on squared paper as shown in Fig. 7. Further examples should also be worked on squared paper.

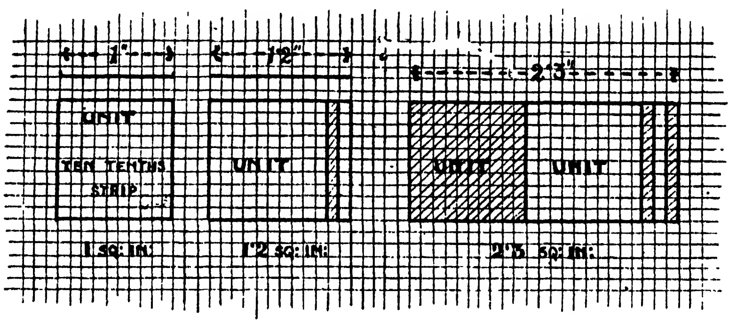


FIG. 7

2. ADDITION OF DECIMALS.

(1) Line.

Tell children to draw two separate lines, one an inch long, the other 1 decimal 4 in. long (as in Fig. 8), and mark the tenths in the longer one. Now draw the two lines as one, measuring each off separately, as in Fig. 8. Measure the new line and find it to

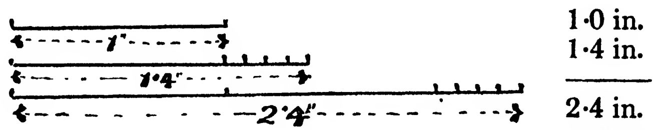


FIG. 8

be 2·4 in.; mark this length underneath. Make a sum of these two lengths, and show the result the same.

Repeat this or a similar sum with narrow strips of paper.

Take another example. Draw two lines—1 decimal 5 in. and 1 decimal 8 in. long respectively (Fig. 9). Draw the same lines

end to end, having the tenths in the middle (Fig. 9). Measure this new line—found to be 3·3 in.

Children will perhaps enquire about "carrying" of tenths to the units' line. If so, refer to fact that 10 tenths = 1 unit. Demonstrate this with line or strips, or with square inch and 10 tenth strips.

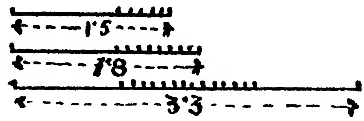


FIG. 9

Refer to the number of units in tens and the number of tenths in a unit. Then over and above each whole unit there are ·5 in. and ·8 in., or 5 tenths and 8 tenths, which together make 13 tenths. This is equal to 1 unit and 3 tenths left. Hence the answer. Let children measure and count the tenths, marking off every 10 tenths as a unit.

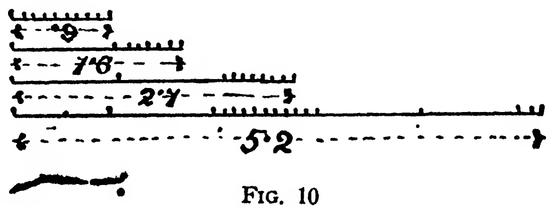


FIG. 10

Consider a more difficult example—three lines. Tell children to draw three separate lines, 0·9 in., 1·6 in., and 2·7 in. (Fig. 10). Mark them each off into units and remaining tenths.

Draw the same lengths end to end (Fig. 10) as before, and measure the new line; this, of course, is 5·2 in. Make a sum of the three numbers and verify the answer.

At this stage the children might be allowed to express tenths either as decimals or fractions. Later on, the use of decimals should be insisted upon.

0·9 in.
1·6 in.
2·7 in.
5·2 in.

(2) **Area.**

Let children cut out 6 or 7 square inches, in thin cardboard, and divide three of them into 1 in. strips (as in Fig. 4). Then work out

sq. in.
1·0
1·4
2·4

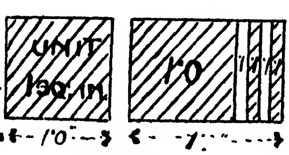


FIG. 11

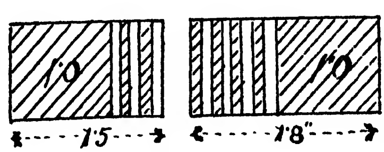
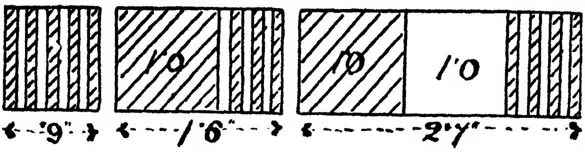


FIG. 12

sq. in.
1·5
1·8
3·3

the same sums that were given with the line (Figs. 11 and 12). Use a square inch as the unit and as many of the tenth strips as are required in each case.

Build them up in line or column (Figs. 11, 12, 13); arrange to displace 10 tenth strips for a unit inch, count up the number of each, and measure the whole. Make sums on them as before.



sq. in.
0·9
1·6
2·7
5·2

FIG. 13

Paste these examples in the exercise books, underneath, if possible, the previous examples, so that the three ways of working out may be shown, viz., line, strip, and square inch.

Problems and Exercises—I.

1. Draw lines, cut strips and areas to represent 1·9 in., 2·5 in., 3·7 in. Say how many tenths there are in each case.
2. How many tenths are there in an inch ? Show this in line and area. Give another name for 5 tenths and 2 tenths.
3. Write down in two ways the numbers : five decimal seven, and six and nine tenths.
4. Cut out a paper which is 3·5 in. wide and 5·0 in. long. Find the number of whole square inches and half square inches in its area, and its total area.
5. If a square has a side 4·7 in. long, find the length of two sides, three sides, and the distance round.
6. A table is 4·5 ft. wide and 6·0 ft. long. Find the length of two long sides and also of two short sides. Also the perimeter.
7. Draw a line about 10 in. long. Divide it into four unequal parts at random. Measure each part and find the total length of the line. Verify your answer by measuring the whole line.
8. Add together 3·2, 5·3, 17·1, 25·8, and 9·6.
9. If it takes 6·4 ft. of cloth to make a pair of trousers, 3·9 ft. to make a waistcoat, and 4·7 ft. to make a coat, how much stuff shall I want to make the whole suit ?
10. How much material shall I want to make 5 waistcoats, 3 coats, and 6 pairs of trousers ? (See previous Question.)
11. One boy runs 306·8 yds., another 125·6 yds., another 84·7 yds., and another 53·4 yds. What is the distance travelled by all the boys ?
12. If I have 3·7 shillings, Tom 5·9 shillings, George 8·1 shillings, and Jack 2·5 shillings, how much have we altogether ?
13. Form a frame of paper 5·6 in. long and 4·3 in. wide, and 1 in. thick all round. How far is it round the outside ? And the inside ?
14. Make a letter H with square inches and tenth strips, 3·8 in. high and 3·0 in. wide. How far is it round ?

15. A letter  is 4·3 in. high and the top is 3·9 in. long, how many *whole* square inches will be needed to make it ? Find also the total number of square inches necessary.

16. A right-angled triangle has its two short sides 1·9 in. and 2·7 in. long. Find the length of the third side ; also its perimeter. (Draw and then measure.)

17. Cut out any triangle of which one side is 2·1 in. long ; find the length of the other two sides and its perimeter.

18. From a 4 in. square cut three triangles ; find the length of all the sides and the perimeter of each triangle.

19. A man gave away 2·3 pence, 4·5 pence, and 7·3 pence ; and if he had 9·9 pence left, how much had he at the start ?

3. SUBTRACTION OF DECIMALS.

(1) Line.

Much the same method should be adopted in dealing with subtraction as was done with addition of decimals.

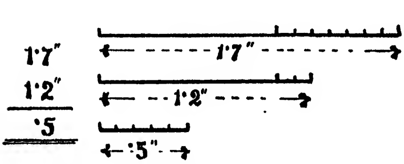


FIG. 14

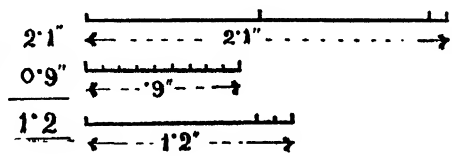


FIG. 15

Take a simple example first, like Fig. 14. Tell children to draw a line 1·7 in. long and another immediately underneath it 1·2 in. long. From their respective positions it will be seen that the first line extends beyond the second ·5 in. or $\frac{5}{10}$ of an inch, *i.e.*, a part equal to the third line, which must be drawn. This, then, represents the difference in length.

Work out in figures thus : 2 tenths taken from 7 tenths leaves 5 tenths, or ·2 taken from ·7 leaves ·5.

Repeat this with the narrow strips of paper, as in addition.

Introduce an example which involves the change of a unit to its equivalent tenths (as in Fig. 15). Insert the "0" to indicate that no whole units have to be subtracted. Here 9 tenths have to be subtracted from 1 tenth—an impossibility. Use one of the whole units composing the 2 units ; that is, leave one whole unit intact and change one to 10 tenths. Now, 10 tenths and the 1 tenth already in the top line make 11 tenths, or the whole top line may be represented by 1 unit and 11 tenths. Take the 9 tenths from the 11 tenths, which leaves 2 tenths ; and nothing to be subtracted from the 1 whole unit leaves altogether 1 unit and 2 tenths, *i.e.*, $1\frac{2}{10}$ or 1·2.

Have the lines drawn (as in Fig. 15). From the first one (2·1 in.) mark off 0·9 in., measure the remainder, and the result will give

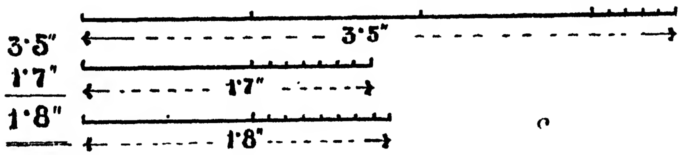


FIG. 16

1·2 in. Keep the ends of the lines level at one end, and mark off the part from the opposite end.

A step further is the one given in Fig. 16, which necessitates subtraction of units from units after conversion of 1 into tenths.

Tell children to draw a line 3·5 in. long and another immediately underneath 1·7 in. long (Fig. 16). Divide the third inch from the left into tenths, and mark off from the second inch the whole inch, and then 7 tenths from the next. Count up what is left when this portion is removed, and we find 1 whole inch and 8 tenths, that is, $1\frac{8}{10}$ in. or 1·8 in.

1·7 in.
1·2 in.

·5 in.

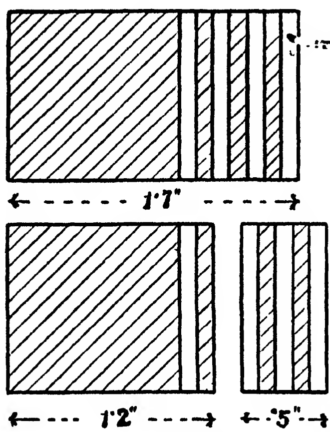


FIG. 17

Work out in sum form as follows: 7 tenths have to be taken from 5 tenths—an impossibility. Hence it is necessary to convert one of the three whole units of the 3·5 into tenths, and that gives 2 units and 15 tenths. Here 7 tenths from 15 tenths leaves 8 tenths, and 1 unit from 2 units leaves 1 unit, so :—

$$\begin{aligned} 3{\cdot}5 &= 2 \text{ units} + 15 \text{ tenths} \\ 1{\cdot}7 &= 1 \text{ unit} + 7 \text{ tenths} \\ \hline 1{\cdot}8 &= 1 \text{ unit} + 8 \text{ tenths} = 1\tfrac{8}{10} = 1{\cdot}8 \text{ in.} \end{aligned}$$

(2) Area.

Proceed with the same sums, using now the square inches and the tenth strips as previously with addition of decimals. Build up the top line of the sum (Fig. 17—1·7 in.) in whole and tenth strips; place immediately under it the second line (1·2 in.); the amount projecting in the top line beyond the second line is the difference (·5 in.).

In the second case (Fig. 18), change one of the unit squares into 10 tenths; take away the 9 tenths from the 11 tenths (2 tenths + 9 tenths), and that leaves 2 tenths; having no units to subtract, one remains, thus:—

$$\begin{aligned} 2{\cdot}1 &= 1 \text{ unit} + 11 \text{ tenths} \\ 0{\cdot}9 &= 0 \quad ,, \quad + \;\; 9 \quad ,, \\ \hline 1{\cdot}2 &= 1 \text{ unit} + 2 \text{ tenths.} \end{aligned}$$

2·1 in.
0·9 in.
1·2 in.

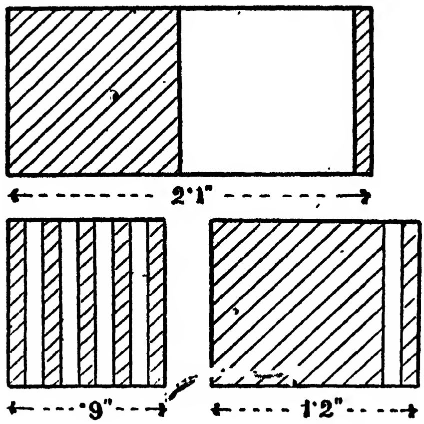


FIG. 18

In the next example (Fig. 19) the same reasoning holds good. Change 1 unit into tenths and proceed as before.

Further exercises should be given in this graphic addition and subtraction. This should be further supplemented with work on squared paper.

Paste all these examples in the exercise book uniformly with the previous ones on addition.

3·5 in.
1·7 in.
1·8 in.

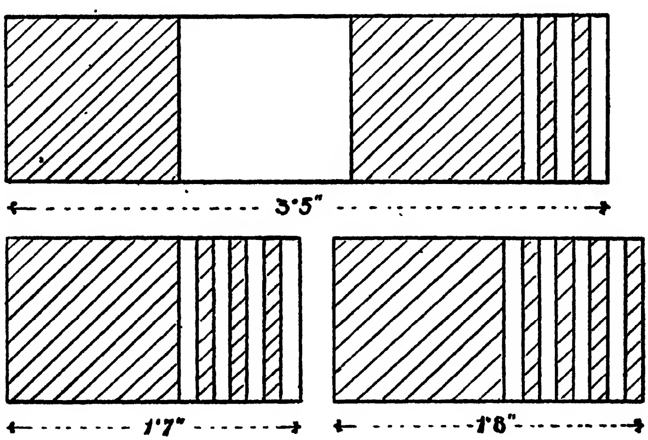


FIG. 19

Let the children measure up books, pens, pencils, rubbers, desks, and papers in inches and decimals; have these written down and sums based upon them.

4. HANDWORK EXERCISES.

Letter Wrapper (Fig. 20). From a piece of brown or drawing paper cut out an oblong 6·5 in. × 9 in. Rule up according to dimensions shown. Sketch the curve at the top, and cut away

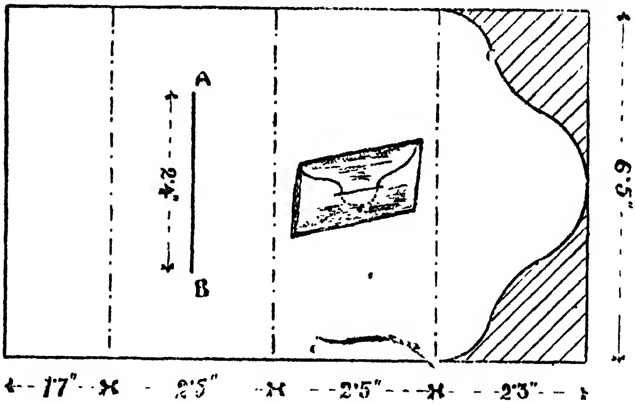


FIG. 20

the waste material shaded. Make a slit in the middle of the second section. Fold the bottom section inwards along the drawn line. Crease the others in the same way. The slit is for the rounded top to be pushed into when folded up.

Paper Bag (Fig. 21). Use fairly thin paper for this model, and rule up as shown in (*a*) and (*b*); cut away the shaded portion in (*b*); fold the sides over to the middle, and where they overlap

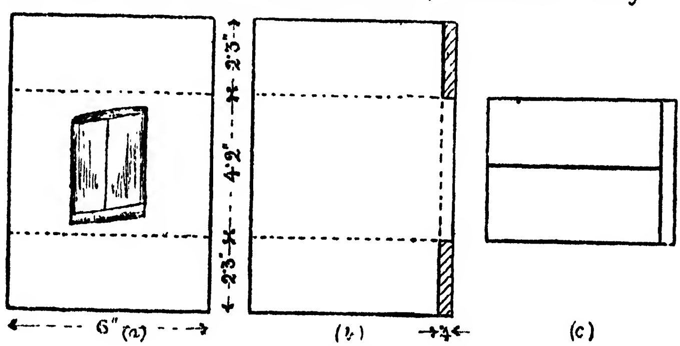


FIG. 21

(·4 in.) paste down, being careful not to paste the bag down the middle of the inside. Fold up the bottom flap and paste that down.

Seed Packet (Fig. 22). For this model use a paper 6 in. × 6 in., similar to that of the preceding exercise. Rule up according to

dimensions shown; cut away the shaded portions, and fold along the dotted lines. Paste down the side flap first and afterwards the bottom one.

Problems and Exercises—II.

1. Cut out a paper oblong 4·7 in. long and 3·3 in. wide; how much longer is it than wide? Find its perimeter (length of long sides and short sides together).

2. If a stick is 3·8 ft. long, how much must be added to it to make it 5 ft. long, and how much cut off to make it 2 ft. long? Cut strips to prove this.

3. My pencil is 5·5 in. long; Tom's is 4·7 in. long; how much is mine longer than Tom's?

4. One oblong paper is 7·3 in. × 4·9 in., another is 5·7 in. × 3·8 in. How much is one perimeter more than the other? How much is one oblong shorter than the other?

5. Draw two lines any length and find how much one is longer than the other. Also how long the two are together.

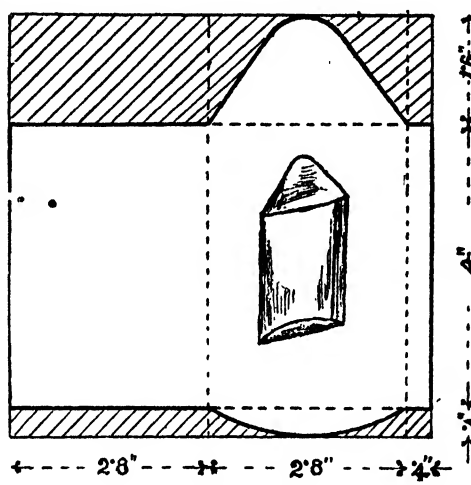


FIG. 22

6. A book is 7·5 in. long; if another is 2·6 in. shorter, how long is the latter?

7. If it takes 6·5 ft. of cloth to make a pair of trousers and 3·8 ft. to make a waistcoat, how much more stuff will be required for the former than for the latter?

8. A boy runs 125·8 yds. in ¼ min., and another 142·5 yds. in the same time. How much further does one boy run than the other?

9. Suppose Jack has 7·5 pence and Harry has 3·7 pence, how much have they both, and how much has Jack more than Harry?

10. In Fig. 20 how much longer is the paper than it is wide? Find how much more of the paper would have to be cut along the slit to cut it right across.

11. Find in making the Paper Bag (Fig. 21) how wide the overlap is for pasting down the middle. How much longer is the bag than it is wide?

12. Find the length of the Seed Packet (Fig. 22) when it is made; also its width. How far round is it? How much longer than wide?

5. MULTIPLICATION OF DECIMALS.

(1) By a Single Unit.

Draw and cut out an oblong, 2 and 3 tenths inches (2·3 in.) long and 1 in. wide. Rule up into square inches and tenths, as in Fig. 23. Make three strips exactly like it. Find the area of one of them; place three of them together, as in Fig. 23; and note the area by counting square inches and tenths. Make an addition sum and find the area. Call attention to the fact that there are three oblongs exactly alike, namely, 1 in. wide and 2·3 in. long. Suggest a shorter way of finding the area of the large oblong by multiplying the small oblong by 3:—

$$\begin{array}{r} 2{\cdot}3 \\ 2{\cdot}3 \\ 2{\cdot}3 \\ \hline 6{\cdot}9 \\ \hline\hline \end{array} \qquad \begin{array}{r} 2{\cdot}3 \\ 3 \\ \hline 6{\cdot}9 \\ \hline\hline \end{array}$$

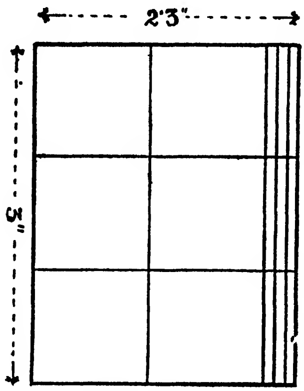


FIG. 23

$$\begin{array}{rcl} 2{\cdot}3 & = & 2 \text{ units} + 3 \text{ tenths} \\ 3 & & 3 \\ \hline 6{\cdot}9 & = & 6 \text{ units} + 9 \text{ tenths.} \end{array}$$

Take another example. Cut out an oblong 1 in. wide and 3·2 in. long. Find its area by ruling up and counting as before. Place four of these strips together to form Fig. 24. Find the area of new oblong by counting and addition. Multiply as in previous exercise. Four times 2 tenths = 8 tenths, and four times 3 units = 12 units.

$$\begin{array}{r} 3{\cdot}2 \\ 3{\cdot}2 \\ 3{\cdot}2 \\ 3{\cdot}2 \\ \hline 12{\cdot}8 \\ \hline\hline \end{array} \qquad \begin{array}{r} 3{\cdot}2 \\ 4 \\ \hline 12{\cdot}8 \\ \hline\hline \end{array}$$

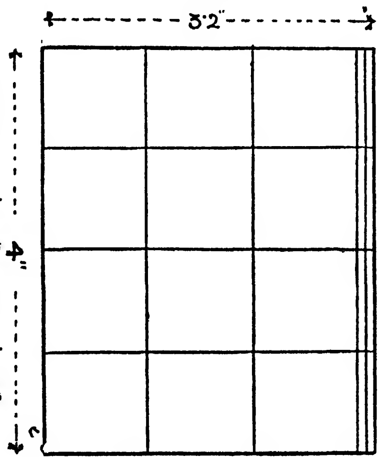


FIG. 24

$$\begin{array}{rcl} 3{\cdot}2 & = & 3 \text{ units} + 2 \text{ tenths} \\ 4 & & 4 \\ \hline 12{\cdot}8 & = & 12 \text{ units} + 8 \text{ tenths.} \end{array}$$

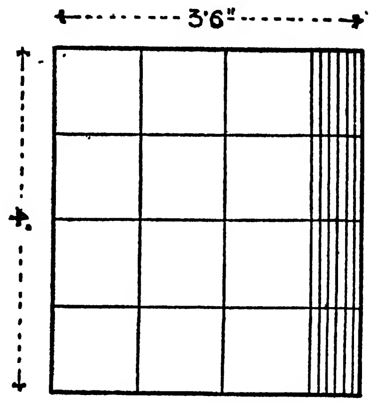
(2) By a Single Unit involving "Carrying."

Build up Fig. 25 as before, in four rows of 3 units and 6 tenths each. Find the area of one row, count up the number of tenths employed altogether, and ascertain the number of whole

units and tenths this is equivalent to (2 units + 4 tenths); add these two parts together, and obtain total result (14·4).

$$\begin{array}{r} 2{\cdot}4 \\ 12{\cdot}0 \\ \hline 14{\cdot}4 \end{array}$$

Set down 3·6 (area of one row) four times and add; same result. Set down 3·6 and multiply by 4. Now 4 times 6 tenths gives 24 tenths, or 2 units and 4 tenths. Set the 4 tenths down under the 6 tenths in the multiplicand and "carry" the two units; 4 times 3 units gives 12 units; + 2 units carried, total 14·4 units.



$$\begin{array}{r} 3{\cdot}6 \\ 3{\cdot}6 \\ 3{\cdot}6 \\ 3{\cdot}6 \\ \hline 14{\cdot}4 \end{array} \qquad \begin{array}{r} 3{\cdot}6 \\ 4 \\ \hline 14{\cdot}4 \end{array}$$

FIG. 25

$$\begin{array}{rcl} 3{\cdot}6 &=& 3 \text{ units} + 6 \text{ tenths} \\ 4 &=& \qquad 4 \\ \hline 14{\cdot}4 &=& 12 \text{ units} + 24 \text{ tenths} \\ &=& 12 \text{ units} + 2 \text{ units} + 4 \text{ tenths} \\ &=& 14 \text{ units} + 4 \text{ tenths.} \end{array}$$

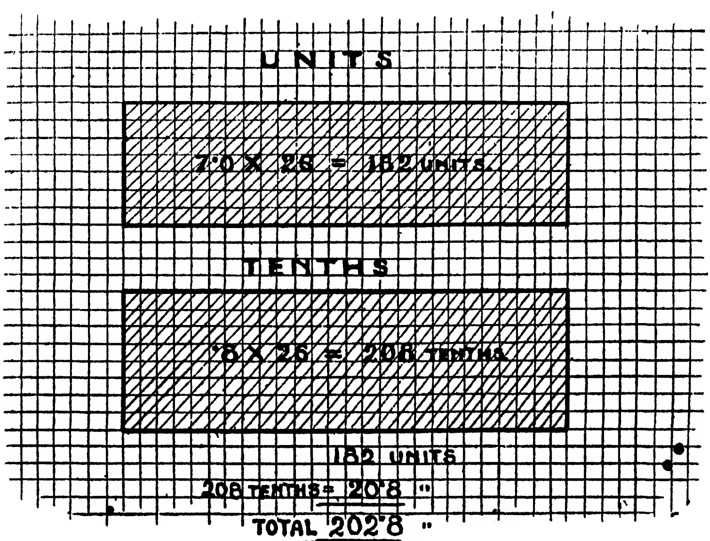


FIG. 26

Verify by other simple examples worked practically with lines, areas in paper or card, as above, and on squared paper.

Paste all together in order in exercise book for future reference.

(3) **By Two Figures involving "Carrying."**

Multiply 7·8 in. by 26. Work as two sums, namely, 7·0 × 26 + ·8 × 26. Work on squared paper; draw an oblong 7 × 26 and find the area. Let this represent the units (Fig. 26). Draw another ·8 × 26 and find its area. Now resolve the tenths into their equivalent units, viz., 20·8 units. Add to the whole units and obtain the total, 202·8 units.

7·8	
26	
156·0	= 20 times
46·8	= 6 ,,
202·8	= 26 ,,

Work now as a multiplication sum. Set down, as in Part II, § 8, and multiply in the usual way by 20 first and 6 afterwards. (Note that the first line is 20 times the top line and not twice.)

Give further exercises of this kind, and let children measure up papers, books, cards, slates, etc., and find their area.

6. HANDWORK EXERCISES.

Needle Case (Fig. 27). Make the Needle Case from a piece of carton paper 5 in. × 6 in. Rule up as shown, and draw in the

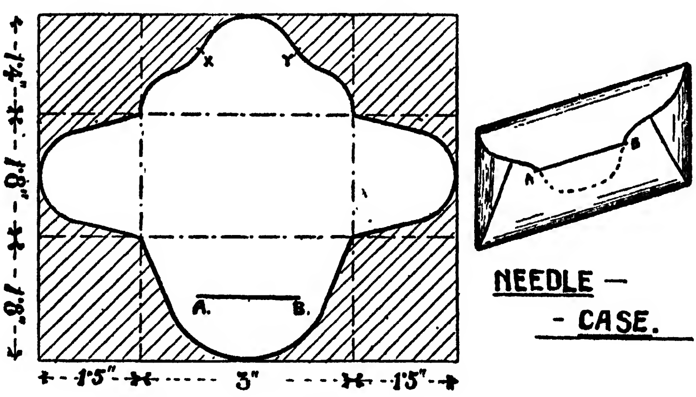


FIG. 27

curves. Cut out the waste material and fold carefully along the lines; the two sides first, then the bottom flap, and on to this fold the top. To make the slit AB, mark from the points X, Y while folded down.

Match Stand (Fig. 28). Cut out two pieces of thin cardboard, one 2 in. wide and 4·3 in. long, the other 2 in. square. The former rule up as shown in the diagram, half cut (or score with the edge of the scissors) along the drawn lines, and fold away from the half

cut. Cut out the shaded portions in order to make the flanges (*f*) fit on the inside when fastened to the square base. Fasten the side flange (*f*), and fasten the whole on to the base in the centre

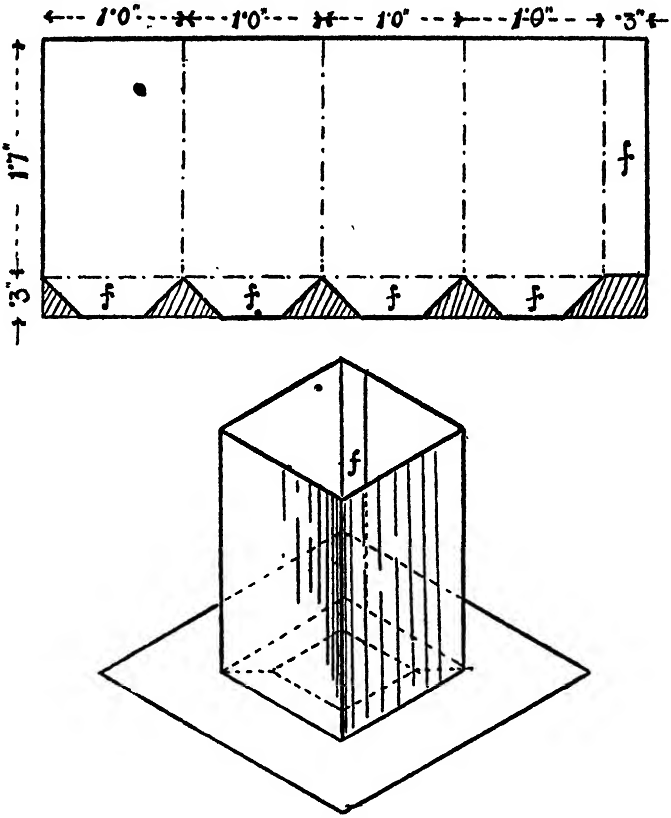


FIG. 28

with Seccotine or fish glue. See that the flanges on the inside are properly "mitred," in order to form a complete right angle when fixed.

Problems and Exercises—III.

1. Find the area of a book 7 in. long and 4·6 in. wide. Cut a paper out one quarter its area and find its perimeter.

2. A room is 12 ft. long and 8·6 ft. wide. How far is it round the room and what is its area?

3. In the Paper Bag (Fig. 21) find the area of each of the oblongs ruled in (*a*); also the area of the flange in Fig. 22 and the area of the packet when finished.

4. If a postage stamp is a square inch in area, how many can I get from a sheet 6·5 in. long and 4 in. wide?

5. What is the area of a piece of note-paper if it is 4·9 in. wide and 6 in. long? What are the length, width and perimeter of an envelope half this area?

6. In the Needle Case (Fig. 27), what are the area and perimeter of the finished model?

7. Find the area of each piece of cardboard used in Fig. 28; also the area of one side, two, and four. How much farther round the base is it than round the sides?

8. If the blackboard is 5·6 ft. long and 3 ft. wide, find how many square feet it contains.

9. I bought a piece of cloth 36 ft. long and 2·5 ft. wide; how many square feet did I get?

10. A tailor uses 12·5 sq. ft. in making a suit; if he makes 8 suits, how much cloth does he use? How much will they all cost at 10s. each?

11. Tom has 24·2 pence; if he gives 4·5 pence to Jack and twice as much to Harry, how much does he give away and how much has he left?

12. If I draw a line 3·4 in. long, what length will one 27 times as long be?

7. Division of Decimals.

(1) Divide 6·4 by 2.

Let the children draw an oblong 2 in. × 3·2 in., and mark it up into square inches and tenths, as in Fig. 29. Find the area in square inches. Fold across the long way of the paper; note the area now—3·2 sq. in.: multiply this by 2 and prove.

$$\begin{array}{r} 2\underline{)6{\cdot}4} \\ \underline{3{\cdot}2} \end{array}$$

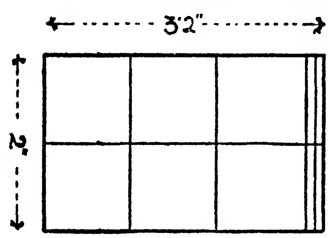


FIG. 29

Set down in the usual way and divide by 2. In 6 units there are 3 twos—set down 3 under the 6. In 4 tenths there are 2 twos—set down 2 under the 4 tenths. Then 6·4 ÷ 2 gives 3·2.

Show that this result is the same as the one obtained in the cutting of paper.

(2) Divide 13·6 by 4.

Tell the children to cut out an oblong 3·4 in. × 4 in.; rule up into square inches and tenths, as in Fig. 39, and find the area of the whole oblong by counting and multiplication.

Fold into quarters across the paper, and find the area of this by measurement, etc.

Work in the usual way by division by 4. There are in 13 units, 3 fours and 1 unit left. Set down the 3 under the 3 and place the

decimal point in the quotient. It is impossible to divide the 1 unit further, so it is converted into tenths (10 tenths = 1 unit). These 10 tenths + 6 tenths to be divided = 16 tenths; 16 tenths ÷ 4 = 4 tenths. Place the 4 under the 6 of the dividend.

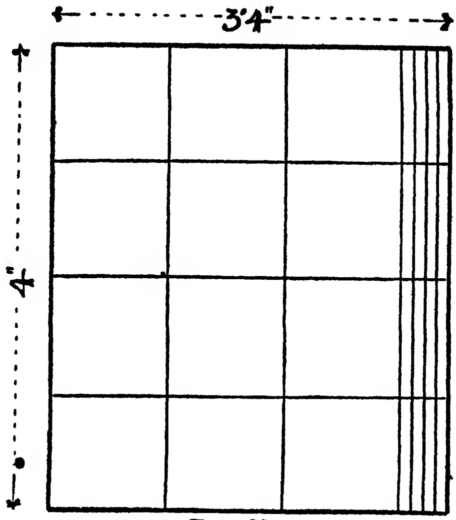


FIG. 30

$$\frac{4)13{\cdot}6}{3{\cdot}4} = \frac{4)12 \text{ units} + 16 \text{ tenths}}{3 \text{ units} + 4 \text{ tenths.}}$$

Compare this result with the practical exercise, and note down in books.

(3) Divide 34·2 by 6.

Give the children six-inch squares of cardboard. Cut off a strip ·3 in. wide down one side. This leaves an oblong 6 in. × 5·7 in.; have this divided into square inches and tenths, as shown in Fig. 31.

Find the area of one cross-strip, and multiply by 6 to verify the area.

Set down in the usual way and divide 34·2 by 6. In 34 there are 5 sixes and 4 units left. Set down 5 under the 4. The 4 units left over, when converted into tenths, give 40 tenths; these 40 tenths + 2 tenths to be divided = 42 tenths; 42 tenths ÷ 6 = 7 tenths. Place the 7 under the 2 after the decimal point.

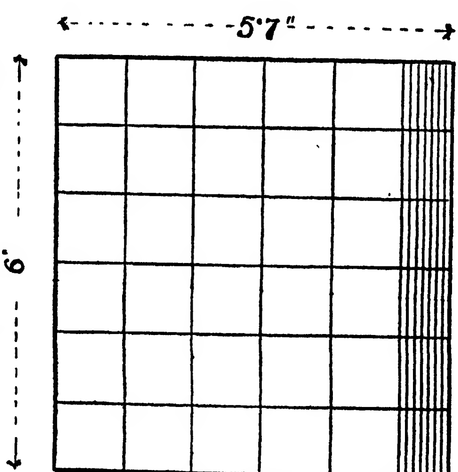


FIG. 31

$$\frac{6)34{\cdot}2}{5{\cdot}7} = \frac{6)30 \text{ units} + 42 \text{ tenths}}{5 \text{ units} + 7 \text{ tenths.}}$$

Notice that again this result is the same as the practical exercise.

Give further examples of this kind and problems in connection with things within the children's capacity.

8. HANDWORK EXERCISES.

Envelope (Fig. 32). From a piece of drawing paper 11 in. ×

7 in. make an oblong 11 in. × 6·6 in. Rule up, as shown in the diagram, and cut away the shaded portions. See that measuring

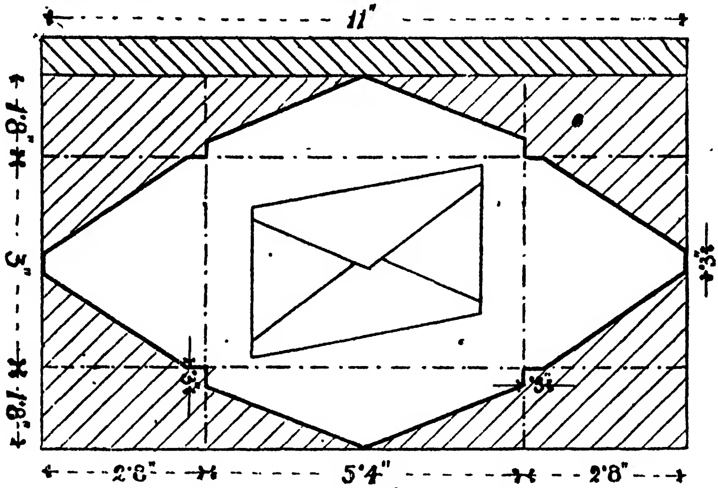


FIG. 32

and ruling are accurately done, or the envelope will not be truly oblong when finished. Fold along the drawn lines and paste together, sides folded inwards first, then bottom folded upwards.

Stool (Fig. 33). Use thin cardboard or stiff paper for this model. Rule up as shown in diagram, and half cut or crease along the

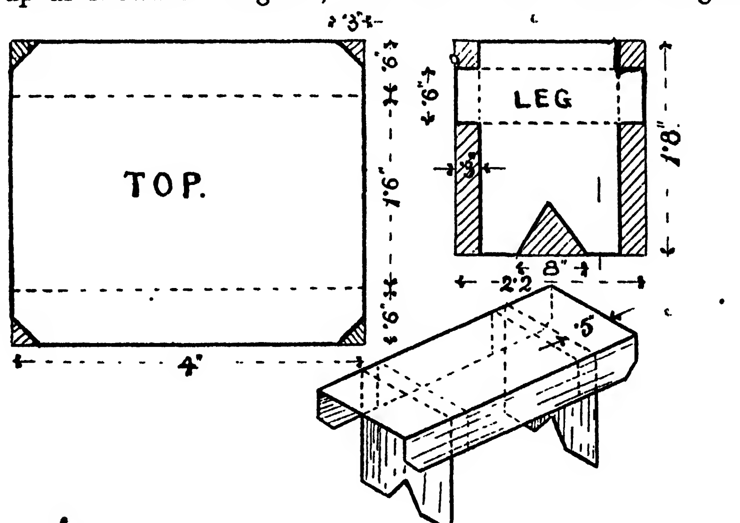


FIG. 33

dotted lines. Cut out the shaded parts, and fasten together as indicated in the general view.

Problems and Exercises—IV.

1. If I have 6·4 yds. of rope and divide it into four equal parts, how long will each part be?
2. The area of an oblong is 9·6 sq. in., and one side 6 in. long; how long is the other?
3. Tom has 13·5 shillings, which he divides equally between Fred, Harry, and Dick; how much do they each get?
4. A plank is 12 ft. long and 2·6 ft. wide. How many boxes can I make from it, if it takes 3·9 sq. ft. for each box?
5. A man had £120, and he gave £15·6 to his wife and divided the remainder equally among his four children; how much did each get?
6. If a postcard is 5 in. long and 3·2 in. wide, how many cards can I obtain from a piece which is 24 in. long and 20 in. wide?
7. The length of a room is 26 ft. and its area 286 sq. ft.; how wide is it?
8. Find the area of the envelope when finished in Fig. 32. Also of the top of the stool in Fig. 33.
9. A book has 48 pages, and the area of one page is 25·2 sq. in. What is the area of all the pages?
10. The length of a page in a book is 7 in., and the area is 32·2 sq. in. What is the width? If there are 120 pages in the book, how far would they reach when placed (*a*) end to end, (*b*) side by side?
11. Show by a drawing that when you divide 15·6 by three the result is 5·2. Prove your answer by multiplication.
12. Jack has 9·8 pence, Tom twice as much, and Harry three times as much. They decided to put it all together and divide it equally. How much did they have each?

9. TRIANGLES.

(1) Properties.

Tell the children to cut out triangles in thin cardboard of various shapes:—

(*a*) All sides equal.
(*b*) Two ,, ,,
(*c*) All ,, unequal.

No dimensions should be given, as long as the conditions laid down are fulfilled. If the desired result is not obtained in each case, further manipulation should be made until the result aimed at is obtained.

Have the sides measured in inches and decimals (Fig. 34) and noted down in books. Find the perimeter in each case, and work sums on the same. Show the perimeter in length of line also.

Place the three triangles side by side, and observe their similarity and dissimilarity.

Each triangle has 3 sides and 3 angles.

One triangle has all its sides equal, another two sides equal, and the other all sides unequal.

Give the names of each of these: *Equilateral*, *Isosceles*, and *Scalene*. Have others sketched in book and named according to the properties of each discovered.

Revise previous lesson on triangles (according to their angles, Section 9, Part II). Compare and contrast these (Fig. 36, Part II) with Fig. 34 below. Note that in this figure all the triangles are *acute angled*, while in Fig. 36, Part II, they are all *scalene*. Examine others and deduce name similarly.

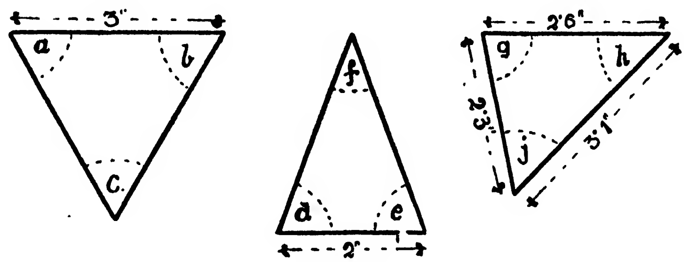


FIG. 34

Have the cut-out triangles copied with rule and compasses into the exercise books. Measure and draw a base line in each case, commencing with the equilateral triangle and making the other two sides equal to the base. In the case of the isosceles triangle, this is only a step further—the length of the two long sides being taken in one operation. In the case of the scalene triangle, it of course necessitates three distinct measurements.

The next step is to have the same or similar triangles constructed geometrically from given dimensions, *e.g.*, those in Fig. 34. Have each one named, the name being written or printed thereon.

Return now to the cardboard triangles cut out earlier; have the angles lettered and cut off along the dotted lines (Fig. 34). Place the angles cut from each triangle together, as indicated in Fig. 35. Notice that in each case a straight line is formed, and

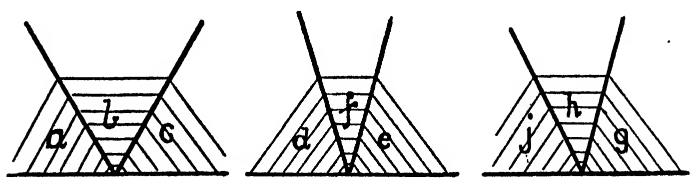


FIG. 35

that the three angles in every case are together equal to two right angles. Test in each case with previously known examples of right angles, like any two taken from a square.

Now, let the children cut off some narrow strips of thin cardboard

—not more than a quarter of an inch wide—3½ in., 2 in., and 1¼ in. long. Let them try to construct a triangle with these strips; they will soon find it is impossible to do so. Ask for suggestions as to what must be done in order to complete the figure. The replies may be either, shorten one or lengthen one sufficiently.

Next let the children place these strips in such a way as to show whether any two strips together are longer than the third. It will be seen in Fig. 36 that, in two of the cases out of the three, different

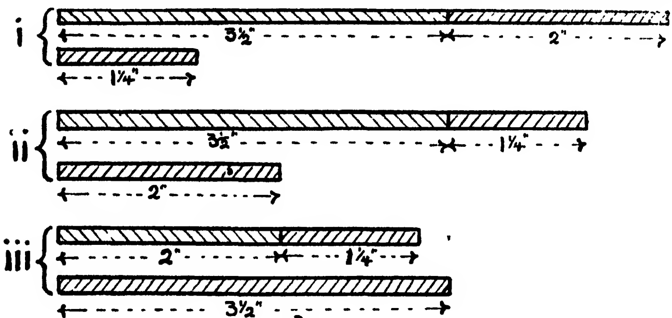


FIG. 36

combinations of sides show that two sides are longer than a third; in the third case (iii) this does not hold good, but the opposite, viz., that one side is longer than the other two.

Still, the triangle from these measurements cannot be constructed. It *must* be shown that *any* two sides are greater than the third.

Construct a triangle with dimensions given in the diagram (Fig. 37), and show by length of line that this triangle follows

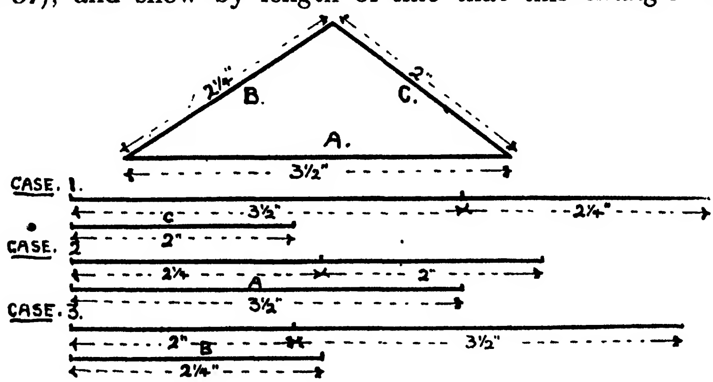


FIG. 37

the law that *any* two sides are together greater than the third side. Make sums, and show the same principle arithmetically.

(2) Set Squares.

As the use of set squares is so eminently important in all

mechanical drawing and in mathematics, it will be well to consider them now in their connection with triangles.

Give to each child a 4 in. square of cardboard and another piece large enough to construct a 6 in. equilateral triangle.

First construct an equilateral triangle on a 6 in. base. Find the middle of one of the sides, and join this point with the angle

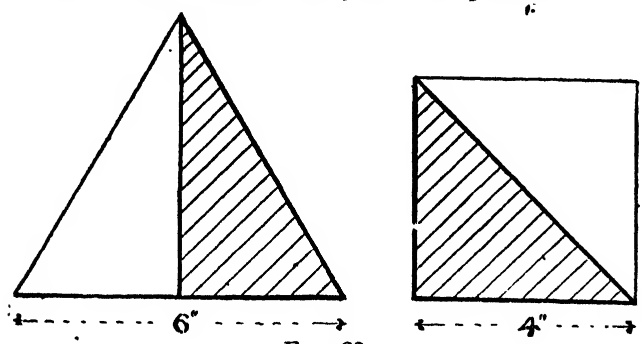


FIG. 38

opposite to it (Fig. 38). Cut along this line; this will give two equal triangles. Examine one of them.

1. Measure sides; they are 3 in., 6 in., and 5·2 in. respectively.
2. Compare the angles; test with the right angle of the square card—one angle is a right angle. Cut an angle equal to the smaller acute angle. Superpose this on the right angle, which is found to be three times as large as the template, the other being twice the size.
3. Hence it is—(*a*) a *scalene* triangle and
   (*b*) a *right-angled* triangle.

Draw a diagonal of the 4 in. square and cut along it (Fig. 38); this will give two equal triangles. Examine similarly to preceding ones.

1. Measure sides; they are 4 in., 4 in., and 5·7 in. (nearly) respectively.
2. Compare the angles, using a template the same size as one of the acute angles.
3. Hence it is—(*a*) An *isosceles* triangle and
   (*b*) a *right-angled* triangle.

For many of the following exercises these cardboard set squares should be used. In fact, as long as they remain usable, it will be best for the children to use them; only when the impracticability of cardboard set squares is discovered should they be discarded and replaced by wooden or metal ones.

(3) To Fold an Equilateral Triangle.

Take a 6 in. square of paper and draw the diameters very carefully. Place in position as in Fig. 38A (*a*), and make the necessary

fold of (*b*), taking care to get a fine, sharp apex, and seeing that the right-hand corner of (*a*) just comes to the diameter already

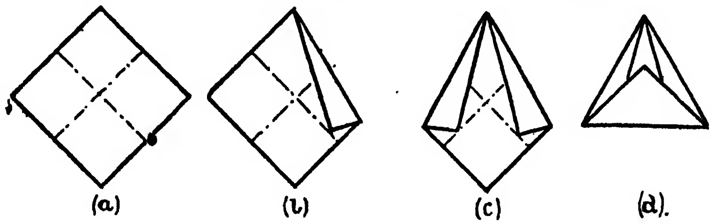


FIG. 38A

drawn. Do the same with the left-hand side, as in (*c*). To complete the triangle, fold up the lower point of (*c*) until each of the bottom corners of the triangle are accurately made. If the creases are well marked, no difficulty will be found in cutting out a true equilateral triangle.

Various measurements may afterwards be made, and with a 6 in. square these work out simply and with no awkward fractions.

10. HANDWORK EXERCISES.

Wall Tidy (Fig. 39). Use thin card for this model, and commence the development by drawing the line A. On the top side of this construct an equilateral triangle, and on the bottom side an isosceles triangle. On sides B and C construct scalene triangles according to dimension given, and on one of the long sides of one of these make a flange (*f*) ·3 in. wide; taper off the bottom of this in order that it will fit when fastened to the opposite side. A hole to suspend the tidy may either be made with the scissors and rounded into shape with a lead pencil, or it may be punched in with a hollow punch.

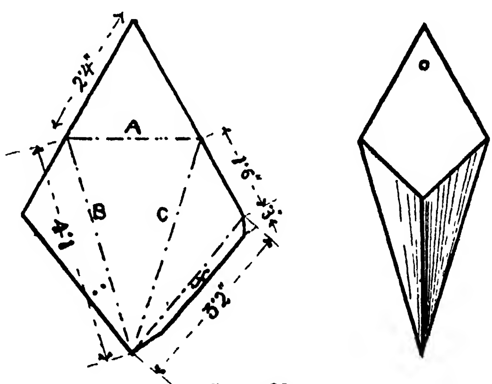


FIG. 39

Photo Frame (Fig. 40). This model is made in three pieces—the front, the back, and the leg. The front is an equilateral triangle of 4-in. side, with an oblong 1·2 in. × 1·6 in. cut out in the middle, ·6 in. from the base of the front. Work this oblong from a centre line down the front. The back is an oblong piece of card (2 in. × 2·2 in.), and fixed on to the back in such a manner as to leave ·2 in. on either side and at the bottom of the aperture

for sticking, and ·2 in. for the photo to slip down. This will leave 2· in. at the top of the card to cover the top of the aperture. The

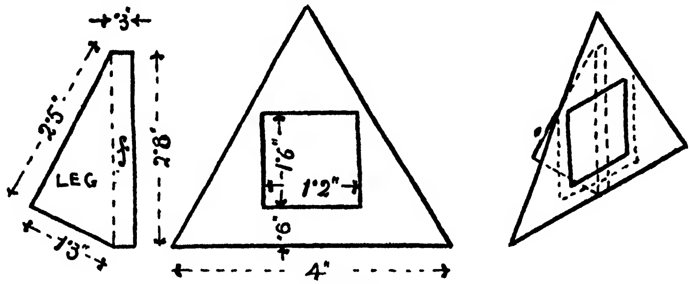


FIG. 40

leg is made as in diagram, the dotted line indicating a half cut for the flange (*f*). This is fastened on to the middle of the back, giving the front the necessary inclination.

Problems and Exercises—V.

1. In Fig. 34 how much farther round the equilateral triangle is it than round the isosceles triangle ? etc.

2. Convert the isosceles triangle (Fig. 34) into an oblong and find its area.

3. Divide the equilateral triangle in Fig. 34 into four smaller similar triangles by joining the middle points of the sides. Prove that the angles are all the same shape.

4. In the isosceles triangle of Fig. 34 show that there are two equal angles as well as two equal sides.

5. In the scalene triangle (Fig. 34) find which is the greatest angle and which the longest side. (Notice that the longest side is always opposite to the greatest angle and *vice versa.*)

6. Find the perimeter of each of the triangles in the Wall Tidy (Fig. 39).

7. In the Photo Frame (Fig. 40) measure the height of the triangular front, and find the perimeter of the half of the front so formed by drawing the middle line.

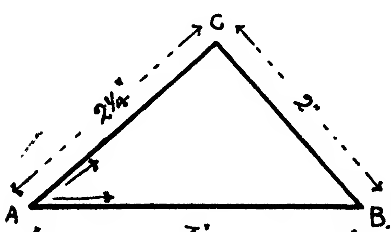


FIG. 41

8. Find the perimeter of the aperture in the Photo Frame and of the leg also.

9. Let the triangle in Fig. 41 form the basis of the following questions ; let each of the sides represent yards or miles, and make sums on the same similar to the following (1 in. to represent 1 mile or foot or yard) :—

(*a*) If I start from A and walk to B, how far do I walk ?

(*b*) If I go through C to B from A, how far do I go? Which is a nearer way?

(*c*) How much nearer is it to go direct from A to B than from A to C and C to B?

(*d*) Find by measuring the lines above and from a cardboard triangle how much farther it is.

10. If a motor-car, in travelling for three hours, goes 18·7 miles, 13·9 miles, and 19·8 miles, how far does it go in the time?

11. If the line C (Fig. 37) is 8 yds. long, how long are B and A?

12. Suppose B (Fig. 37) is 9 miles long, how long are A and C? When each inch stands for 1 mile (foot, yard, etc.), how far is it round the triangle?

11. RECTANGLES.

(1) Square.

Use the cardboard set squares to construct a square on drawing paper. Commence with a base 6 in. long, and at each end draw a line which makes a right angle with the base line (Fig. 42), marking off both 6 in. long. Join their ends and cut out the square.

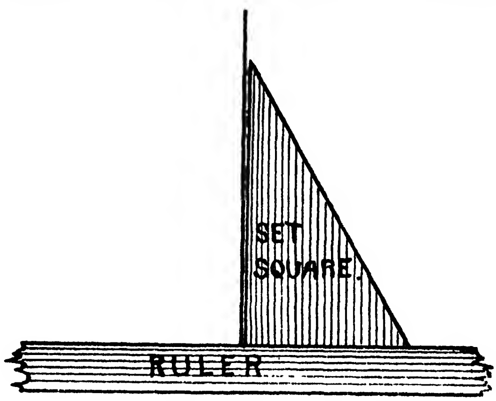


Fig. 42

Revise properties already learned—all sides equal and all angles square or right angles.

Draw the diagonals and diameters of the square (Fig. 43); fold along these lines, and mark the lengths of the diagonals obtained on a piece of drawing paper. Show that they are both the same length. Measure with rule and find both 8·5 in. long.

In the same way deal with the diameter. Use the word "bisect," and let children deduce its meaning. Give the word "perpendicular" to the arms of all right angles—those drawn as above with the set squares are all "perpendiculars."

In noting that the diagonals bisect the square, show also that they bisect each other (Fig. 43), by measuring and marking strips of paper, either half diagonal length or whole length, folding and testing afterwards.

Let the children also note that the diagonals form right angles where they cut each other. Test with known right angles.

Give the children a fresh piece of paper (square), and tell them to find the centre, either by folding or ruling the diagonals or

diameters. Let them see that this point is the same distance from any two corresponding points in the opposite sides, and the same distance from each of the corners. This is why it is called the centre of the square.

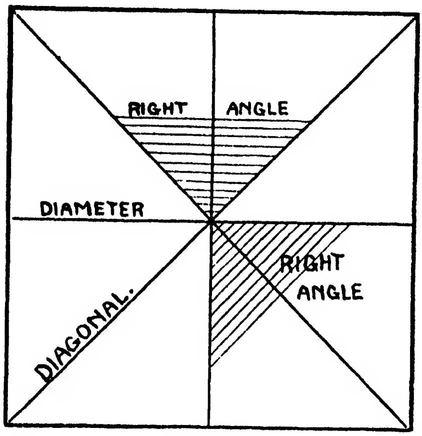


FIG. 43

(2) **Oblong.**

Revise the properties of the oblong already investigated. Treat the oblong in much the same way as the square has been done above, as far as possible :—

1. Diagonals are equal (prove as for the square).
2. Diagonals bisect the oblong (superposition).
3. Diagonals bisect each other (as for the square), but not at right angles (cf. square).

Construct an oblong with set square as for the square. Make the two side perpendiculars at the ends of the base line 4 in. long; the latter is 6 in. long (Fig. 44). Join their ends to complete

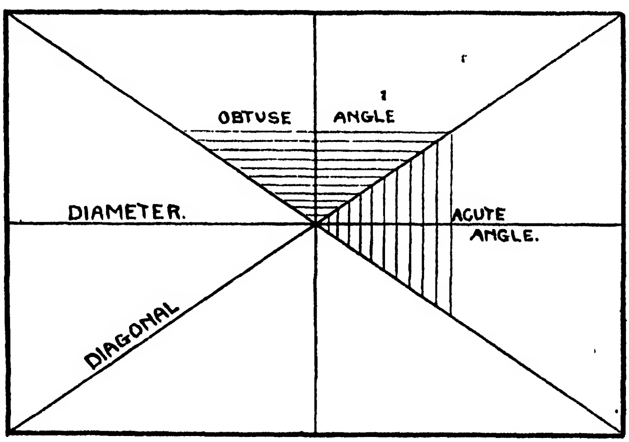


FIG. 44

the oblong, and cut it out. Test in the same way as formerly, and verify the properties stated above.

Give questions on area of the square and oblong; the number of square inches in each, and in half and quarter oblongs and squares. Name the kinds of triangles formed, requiring a reasoned statement with the answers.

12. GRAPHIC REPRESENTATION.

(1) Money.

(*a*) **Shillings and Pounds.** Rule up an oblong 3 in. × 4 in. into square inches, as shown in Fig. 45. Shade in one whole square, half a square and a quarter of a square.

SHILLINGS AND PARTS.

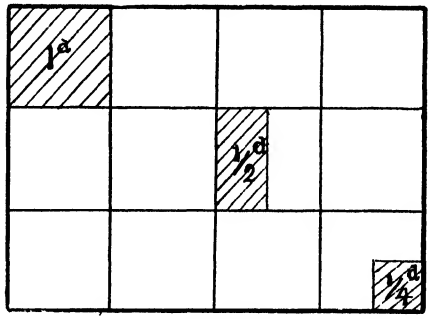


FIG. 45

If the whole oblong represents a shilling, what is one square worth?

Shade in a portion of a square to represent a halfpenny; another to represent a farthing, etc.

Cut up the oblong into pence, halfpence, and farthings; permit children to change pence with one another, and conduct easy buying and selling operations.

Tell children to draw a line 10 in. long, and divide it into 20 equal parts. If this long line represents a sovereign, how much does one small part represent? Draw lines underneath to show half a crown, a florin, sixpence, 5s. 3d., 7s. 6d., 15s. 9d. Add 2s. and half a crown together, and see how long the line is. Add 8s. 3d., 3s. 6d., and 6s. together; mark it out in a line and prove the answer. Show how much of a sovereign is left when 16s. 6d. is taken away. A boy has 15s.; he spends 4s. 3d. on a ball and 9s. 6d. on a bat; how much has he left?

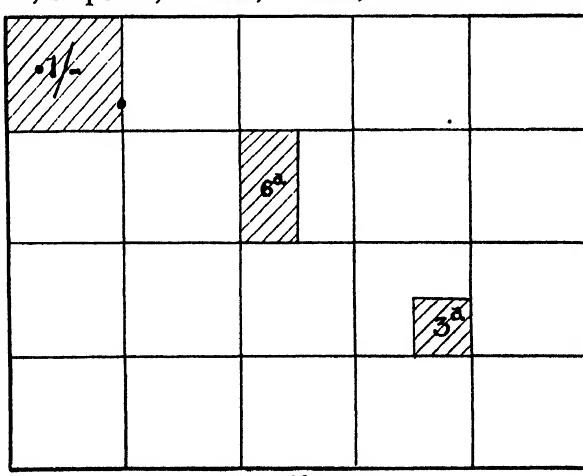


FIG. 46

Draw lines which will show £·2, £·7, and £·9. Add £·1 + £·3 + £1·2. How many shillings is it equal to? If I take £·3 from £1, how much have I left? Draw a line which will show how much is left when 5s. 9d. is taken from 11s. 3d. I had 5s. in my pocket and 1s. 3d. in my purse. I spend 1s. 6d. How much have I left altogether?

Make an oblong 5 in. × 4 in. and rule up into square inches, as in Fig. 46, showing 1s., 6d., and 3d. Cut out the square inches and use as shillings; half square inches for sixpences, etc. If white paper be used for this exercise and brown for the previous figure, then they may be used jointly for practical questions in money values. Shopping and marketing operations should be carried out by the children, the exercises suited to the class of children, whether boys or girls, rural or town. The real operations should be carried out first and written down arithmetically afterwards. Simple reduction should also accompany such exercises: the equivalent in silver to a number of coppers, etc., etc., and *vice versa*.

Various other exercises will suggest themselves to the teacher following on these; the cutting out of pieces to represent the various coins, other sums of money, addition, subtraction, and many other problems both in "line" and "area." A talk here on the advantages of having round coins and paper money will not be out of place.

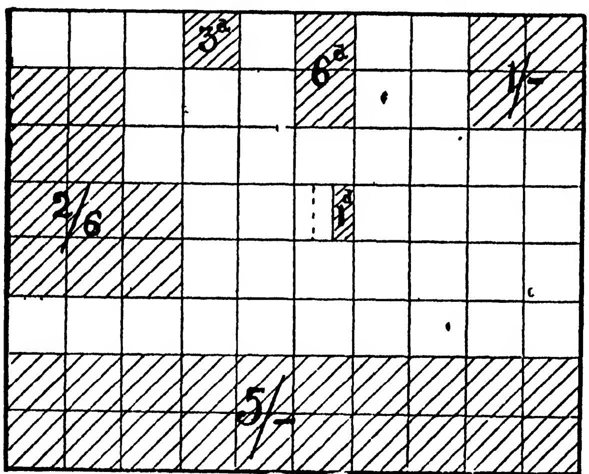


FIG. 47

(*b*) **Comparative Values.** If an oblong 5 in. × 4 in. be ruled up into 80 small squares of half an inch side, as in Fig. 47, the comparative value of various amounts can be graphically shown. Commencing with the whole oblong representing a sovereign, then a quarter (or 20 small squares) of it will represent 5s., and an eighth (or 10 small squares) of it half a crown, etc. Similar problems to those already given should follow to illustrate exercises in reduction, simple fractional parts, etc. Folding and cutting exercises should precede the written and oral work in each case.

(2) Length—Feet and Yards.

(*a*) Revise inches and the foot. Tell the children to cut several strips of fairly stiff paper (drawing paper will do) 1 in. wide and 7 in. long. Paste them in one long strip, overlapping 1 in. for pasting, making a strip long enough to measure three linear feet. Mark

this strip off into feet and inches, and cut off any paper in excess of 36 in.

Indicate clearly the "foot" marks, and give this strip its name —a yard measure.

Tell the children to measure various distances with this strip, and find lengths in yards, feet, and inches, *e.g.*, desk, form, room, playground, easel, blackboard, etc.

Use these measured distances for exercises in reduction from inches upwards and yards, etc., downwards.

Draw a line 4½ in. long (Fig. 48) and divide it into three equal

FIG. 48

parts; let each one of these thirds represent a foot. Divide the first representative foot into 12 equal parts, each one of these then will represent 1 in. Divide the second division into four equal parts; each part will represent 3 in.

Measure off from this scale various lines representing 2 in., 5 in., 8 in., 10 in., 11 in.; 1 ft. 3 in., 1 ft. 9 in., 2 ft. 5 in., 2 ft. 7 in., 3 ft. 1 in., and 3 ft. 6 in.

. Further represent in line distances measured in yards, feet, etc., to scale; awkward examples should be omitted for the present.

(*b*) **Lengths of Rivers** may be represented in line to some simple scale, *e.g.* (Fig. 49), the Thames, Severn, Trent: 2·1 in., 2·2 in., and 2·0 in. respectively, where 1 in. represents 100 miles. Strips of paper pasted in books along with the lines would also enhance the value of the teaching.

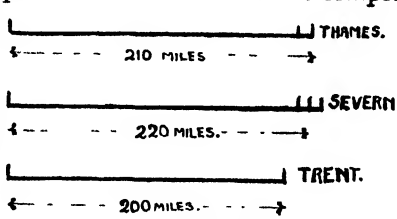


FIG. 49

(*c*) **Heights of Mountains** would be more suitably worked out in thin cardboard (Fig. 50). Commence with some local hill or mountain, and take this as the standard for working other heights. Suppose 1 in. represents 600 ft. in height; then work out the following: Barr Beacon, 900 ft. (1½ in. high); the Peak, 1,500 ft. (2½ in. high); Penygant, 2,100 ft. (3½ in. high); Plinlimmon, 2,400 ft. (4 in. high); Cheviot Hills, 2,700 ft. (4½ in. high); Ben Nevis, 3,000 ft. (5 in. high); Snowdon, 3,600 ft. (6 in. high). Draw a line 7 in. long; let this represent sea level, and each inch the base of the triangle, which is to represent the height of the mountain. Erect a perpendicular on the middle of each one-inch base, and mark off requisite inches. Join, and print in the name and height in feet of each (Fig. 50).

(*d*) **Problems** illustrated in line (Fig. 51) and worked out to scale may also be advantageously given. In the present example,

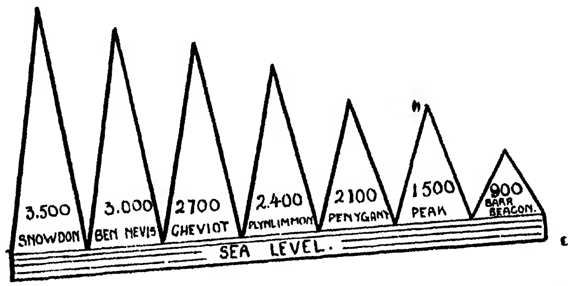


Fig. 50

if from A to B is ¾ of the distance (8 miles) from Walsall to Birmingham, how far is it from B to C, etc.? If I rode ⅝ of the

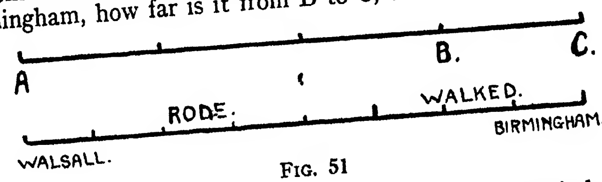


Fig. 51

way to Birmingham from Walsall and walked the remainder, how many miles did I ride and how many did I walk?

Similar "localised" problems will be better, and will suggest themselves to the teacher.

(3) **Time.**

In the same way treat the divisions of time. Tell children to draw a line 6 in. long, and let that represent one day, *i.e.*, 24 hours (Fig. 52). Halve it first to obtain a representation of 12 hours.

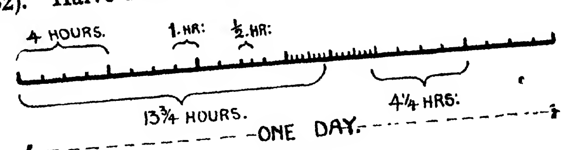


Fig. 52

Next divide the whole line into inches, each representing 4 hours; then each inch into quarters to represent hours. By division of various hours, show half hours and quarters.

Mark off on the line, as shown in the diagram, certain times, *e.g.*, 13¾ hours, 4¼ hours. Draw separate lines showing morning and afternoon sessions of school time to a larger scale; *e.g.*, a 6 in. line may represent the morning session of 3 hours, then a line

$2\frac{1}{2}$ in. long will represent the afternoon session. Have each divided up to represent the duration of the different lessons in both of these sessions. The same may be done for the time

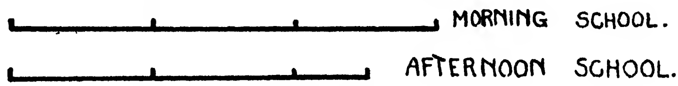


FIG. 53

allotted to the chief lessons during the week. Problems in time on trains, motor-cars, trams, walking, etc., may also be represented in line.

Reduction of time measure graphically expressed will also give children a better idea of the "sense of proportion" of things generally.

(4) **Weight.**

(*a*) **Ounces and Pounds.** Draw and cut out a 4 in. square of paper or thin cardboard (Fig. 54). Let this represent 1 lb. weight. Rule up into 16 square inches; each one of these will then represent 1 oz. Show, by shading in, or cutting out, an oblong to represent $\frac{1}{2}$ oz.; also a triangle to represent the same. Various other weights should also be shown, in a variety of shapes, *e.g.*, $4\frac{1}{2}$ oz. (Fig. 54).

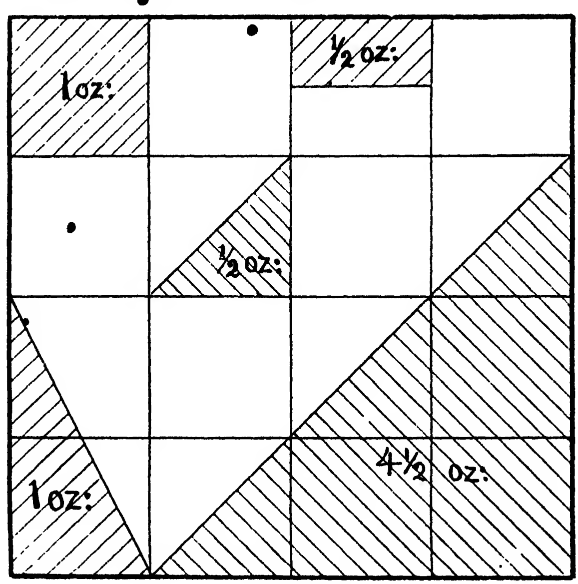


FIG. 54

A line 8 in. long may also be taken to represent 1 lb. and other weights proportionately indicated on it. This will form a good alternative to the preceding exercise, or be supplementary to it.

Show, in "line" or "area," $\frac{3}{4}$ lb., $\frac{1}{2}$ lb., 2 oz., 5 oz., 9 oz., etc. A jar of jam weighs $2\frac{3}{8}$ lb.; the jar itself weighs $\frac{1}{4}$ lb. How much does the jam weigh? If three packets of tea weighed respectively 4 oz., 8 oz., and 12 oz., how much was there altogether?

Simple exercises in reduction from lb. to oz. and oz. to lb. should follow.

(*b*) **Pounds, Stones, and Quarters.** Rule and cut out an oblong 7 in. × 4 in., and rule up into square inches. Each one of these squares will represent a lb. weight where the whole oblong stands for a "quarter" (*i.e.*, a quarter of a hundredweight).

Shade in the various weights shown in Fig. 55, and as far as possible have the same weight expressed in a variety of figures—

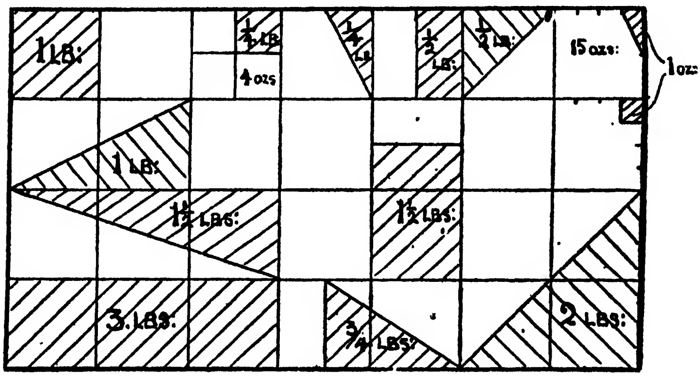


Fig. 55

triangle, oblong, square, and irregular figures. Note also that 14 lb. is called a "*stone.*"

Note the comparative value of the 1 oz. triangle and 1 oz. square, and the other various amounts, with the whole "quarter." Have four of these papers pasted together, and in this way represent a hundredweight. Twenty of these would, of course, represent 1 ton.

16 oz.	=	1 lb.
14 lb.	=	1 stone
28 lb.	=	1 quarter
4 qrs.	=	1 cwt.
20 cwts.	=	1 ton.

Show in line and area various weights including the quarter downwards. Follow with the same sums in reduction, both upwards and downwards, of a practical and familiar nature.

Problems and Exercises—VI.

1. In the middle of a 4 in. line draw a perpendicular 3 in. high; join this with the ends of the base line, and find the perimeter of the triangle. What kind of triangle is it? Give reasons.

2. Draw a 2 in. square, cut it out, and make an isosceles triangle of the same area.

3. The diameters of an oblong are 2·4 in. and 3·0 in. long; draw them first and then construct the oblong, using set squares. What is the area of one of the small oblongs so formed?

4. From Fig. 54 obtain as many different kinds of triangles as you can; copy them from the figure into your books.

5. Why are coins usually made round in shape and of different sizes and material? Why are the edges of some coins "milled" and of others not?

6. If Tom has 3d. for every half-crown I have, how many times more valuable is my money than his? If I have a sovereign, how much has Tom?

7. From London to Birmingham is 120 miles; from Walsall to Birmingham 10 miles; from London to Manchester, 200 miles; from Birmingham to Manchester, 60 miles. Show these distances by drawing lines to scale of ·1 in. to 1 mile.

8. Staffordshire contains 1,000 square miles; Walsall covers 10 square miles. Draw out a square representing the county, and mark on it a space for Walsall.

9. Take any six boys in the class and measure their height. Draw lines to scale of 1 in. to 1 ft. representing the six boys.

10. Draw a line of convenient length representing 12 hours; show by dividing this line the way you spent yesterday. Say, $\frac{1}{2}$ hr. breakfast, $\frac{1}{2}$ hr. going to school, 3 hrs. school, etc., neglecting odd minutes and working in fractions of an hour.

11. Cut out an oblong 3·2 in. long and ·4 in. wide. If this represents 1 lb. weight, show 1 oz., 5 oz., $\frac{1}{2}$ lb., and $\frac{3}{4}$ lb.

12. I bought a sack of flour weighing a hundredweight. I found that the bag weighed 7 lbs.; how heavy was the flour? Show in a line the full sack, and how much was flour and how much sack.

PART IV.

SYNOPSIS.

1. SQUARE MEASURE : AREAS.

(1) The Square Foot. Give each child four 6-in. squares of paper. Have them all ruled up into square inches (Fig. 1), and the area of each and the four together found. Have them joined up into one larger square. This should be done by pasting strips of paper on the back about an inch wide, giving an overlap of half an inch on each adjacent square. When so joined, this large square will be found to be 12 in. along each side. Prove by measuring and folding that it *is* a square, and note its area. Elicit its name and give reasons for it.

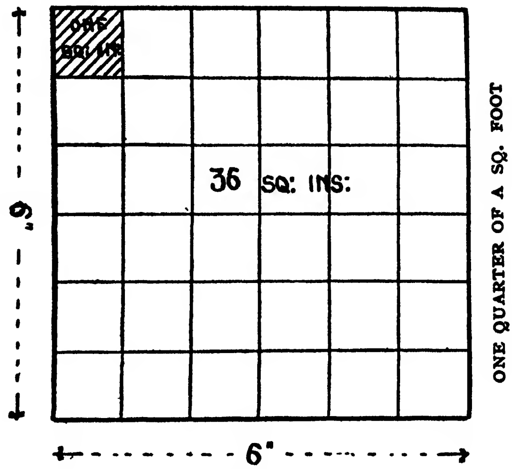


FIG. 1

1 SQ. FOOT = 144 SQ. INCHES.

Refer to the various shapes of a square inch, and apply same

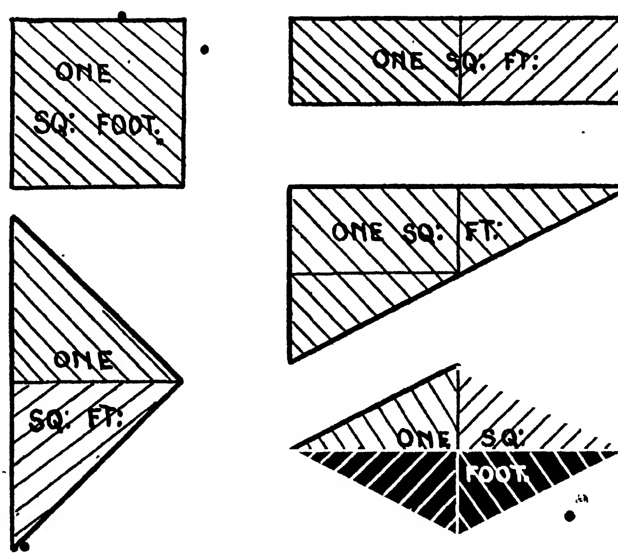


FIG. 2

teaching to the square foot (Fig. 2). Cut out a certain sized square and let this represent a square foot; then cut up in various ways and form different figures of one square foot in area (Fig. 2).

In this last example the children might be allowed to make the representative square (tested, of course, for its properties) of varying sizes, rather than all the same.

(2) **The Square Yard.** For the square yard allow children to work in groups. Tell them that we are going to make a square which is 3 ft. along each side (Fig. 3). The same construction as for the square foot will do very well, or an old newspaper may be utilised—in this case only pasting down the edges of the square foot on to the newspaper as a foundation.

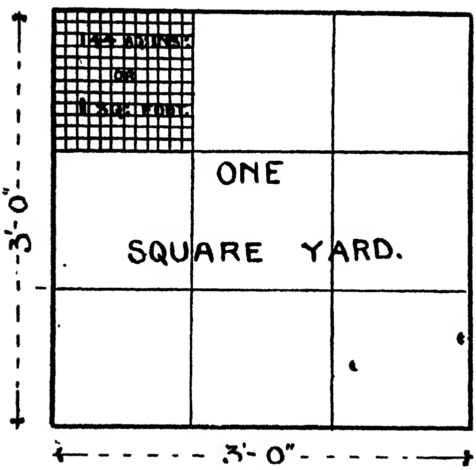


FIG. 3

It will be obvious to all that it requires 9 sq. ft. to make a square of 3 ft. side, or a *square* yard. Hence note the connection between Long Measure and Square Measure, and the latter as resulting from the former.

1. A square on a line 1 in. long = 1 sq. in.
2. " " " 1 ft. " = 1 sq. ft.
3. " " " 1 yd. " = 1 sq. yd.

or

4. A square on a line 12 in. long = 144 sq. in.
5. " " " 3 ft. " = 9 sq. ft.

Here revise that *any shape covering a square foot or a square yard is nevertheless a square foot or a square yard in area.*

144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.

Example (i). Find the area of a box lid 12 in. wide and 16 in. long. First, either supply children with squared paper, or tell them to rule it up into squares of suitable size (Fig. 4). Quarter-inch squares will be found a convenient size, as they are not so small as to be troublesome in counting, and are a good preparation for the much smaller ones necessary in higher stages of mathematics and physics.

Rule up the oblong representing the box lid (each square to represent 1 sq. in.), 16 squares long and 12 squares wide; mark off a square foot, as in Fig. 4; count and calculate the total number of square inches by multiplying the length by the width. Find the number of square feet (that is, the number of groups of 144) in the area and the number of square inches remaining. Compare the working in the drawing with the arithmetical calculation.

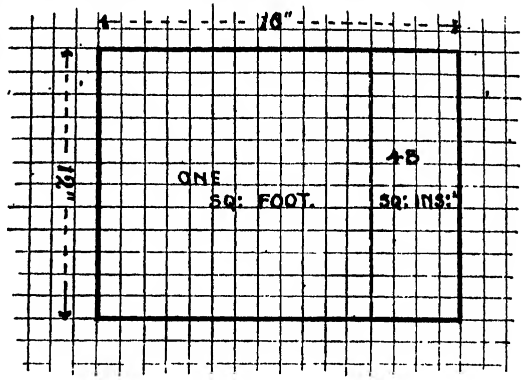


Fig. 4

$$\begin{array}{rl} & 16 \text{ in. long} \\ & 12 \text{ in. wide} \\ \hline 144\left\{\begin{array}{l} 12)192 \text{ sq. in. area} \\ \hline 12)\ 16 + 0 \end{array}\right. & \\ \left.\begin{array}{r} \hline 1 + 4 \end{array}\right\} & 48 \text{ sq. in.} \end{array}$$

Area = 1 sq. ft. 48 sq. in.

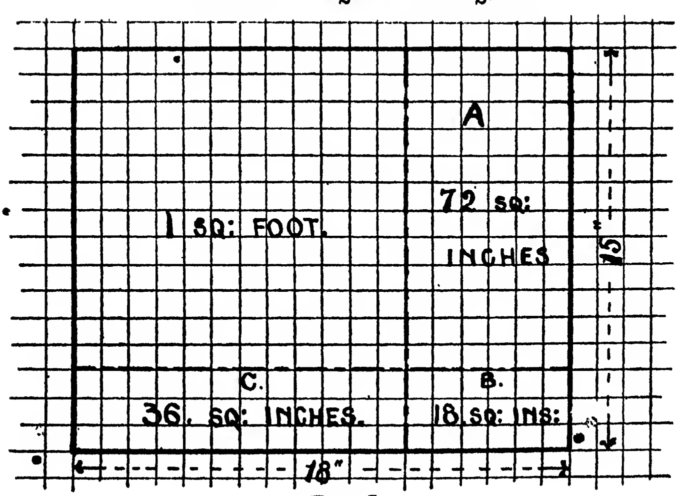


Fig. 5

Cut out an oblong piece of paper to represent the box lid to a scale of half an inch to an inch. Verify the previous methods by calculating the number of small squares that represent a square inch, and the whole square foot shown with the remaining square inches.

Example (ii). Find the area of a drawing board 15 in. long and 18 in. wide. Mark out the length and width of the drawing board on the squared paper to scale, as shown in Fig. 5; also the whole square foot and the three remaining oblongs. Find the number of square inches in each of these oblongs and the total. This will give the area *practically*. Next calculate the area in square inches by multiplying the length by the width. Divide by 144 (the number of square inches in a square foot) and obtain the *arithmetical* answer.

Area of A = 6×12 = 72 sq. in.
,, B = 3×6 = 18 ,,
,, C = 3×12 = 36 ,,
,, A, B, and C = 126 ,,

Total area of the drawing board, therefore, is—

1 SQ. FT. 126 SQ. IN.

$$\begin{array}{r} 18 \text{ in. long} \\ 15 \text{ in. wide} \\ \hline 180 \\ 90 \\ \hline \end{array}$$

$$144\left\{\begin{array}{l} 12)\underline{270} \text{ sq. in. area} \\ 12)\ 22 + 6 \\ \qquad \underline{1 + 10} \end{array}\right. \quad \left.\begin{array}{l} \\ \\ \end{array}\right\} 126 \text{ sq. in.}$$

AREA = 1 SQ. FT. 126 SQ. IN.

Cut out a piece of paper to a scale of half an inch to an inch

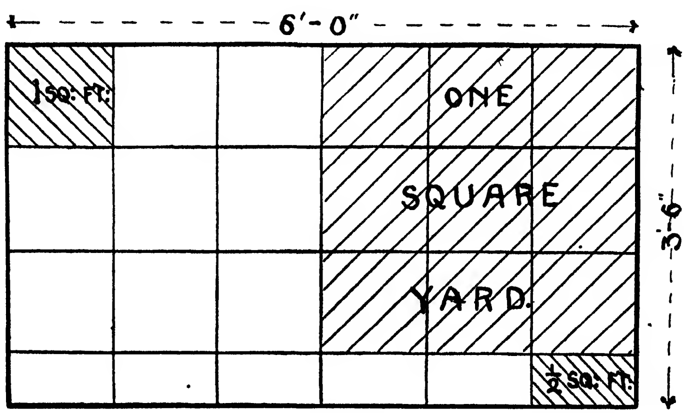


FIG. 6

to represent the drawing board, and verify the answers obtained above. Compare all three results.

Example (iii). Find the area of a table top which is 6 ft. long

and 3 ft. 6 in. wide. Cut out a piece of paper 6 ft. × 3 ft. 6 in. to a scale of 1 in. to 1 foot, as shown in Fig. 6. Rule up into square feet, and find the number of whole square feet, the number of half square feet, and then the area of the whole top. Show the number of square yards also represented in the oblong cut out.

18 whole sq. ft. = 2 sq. yds.	3·5 ft. wide
6 half „ = 3 sq. ft.	6 ft. long
AREA = 2 SQ. YDS. 3 SQ. FT.	9)21·0 sq. ft.
	2 SQ. YDS. 3 SQ. FT.

Repeat the example on squared paper, as in previous examples, and verify the result.

From consideration of the foregoing examples, lead the children to formulate the rule for finding the area of a rectangle. Note that in each case, when working the example arithmetically, the length (in feet or inches) was multiplied by the width *in the same denomination* (feet or inches), and the result obtained was in each case *square* feet or *square* inches accordingly.

Have the rule written down and learned.

$$\text{Length} \times \text{Width} = \text{Area}$$
$$L \times W = A$$

2. HANDWORK EXERCISES.

Picture Frame (Fig. 7). A picture frame measures outside 2 ft. 6 in. long and 2 ft. wide. The width of the moulding is 4 in. Draw a suitable picture[1] to the scale of 3 in. to 1 foot, and construct a framework in cardboard, with the corners properly "mitred." (A mitre is an angle half that of a right angle.) A piece of thin tinted cardboard, large enough for the foundation, should be cut first. On this mount the picture to be framed. Cut out the four pieces for the frame the required shape and size (see that the corners fit nicely), and mount these over the picture. Bind the edges with paper or cloth—the inside of the frame pieces being, of course, done before mounting—or paint a border on the tinted cardboard about a quarter of an inch wide all round. On the back stick with Seccotine or liquid glue a piece of thread for the purpose of hanging the picture up.

Paste or glue pieces of paper over the ends of the thread for about half an inch of its length. It will be as well also to make knots in the ends of the thread, in order that the cotton may not slip through when hung up.

[1] Any suitable piece of brush drawing already executed may be employed for this instead.

Plan of School (Fig. 8). *Scale* ·1 *in. to* 1 *ft.* Use thin straw board for this model, and Seccotine as an adhesive. Thin tinted board may be used if the straw board offers too much resistance for the children. Assuming that the walls are uniformly 1 foot thick, it will be seen that about 500 ft. of this will be required, or actually about 4 ft. of cardboard ·1 in. wide. It will be as well to ignore the spaces for the windows at present, and mark those afterwards with tinted cardboard of another colour; spaces, of course, should be left for the doors. Cut out the base the required size and shape; mark off the classrooms, doors, windows, etc., in pencil, and gradually build up the walls with the strips cut for the purpose. Smaller dimensions, as those of doors and windows, are

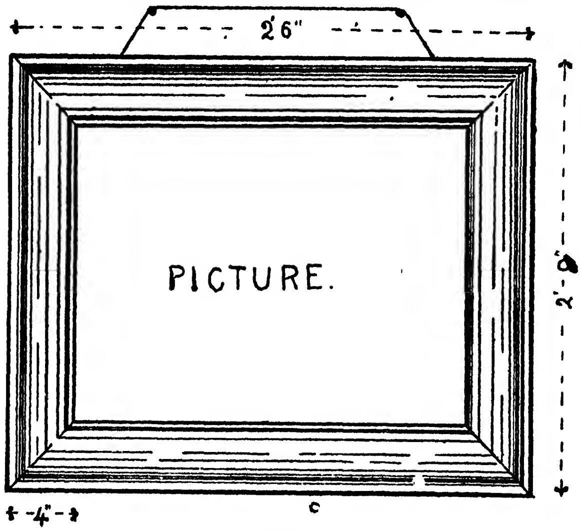


Fig. 7

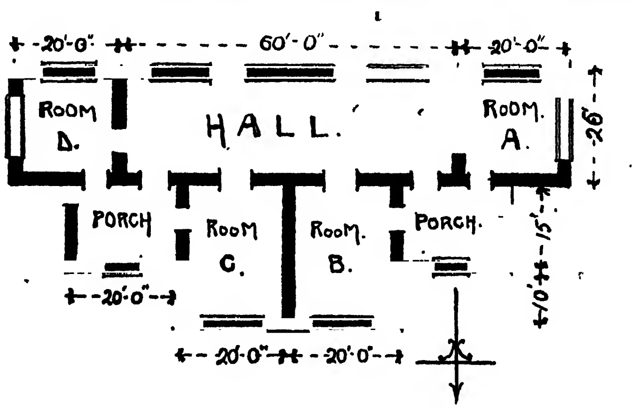


Fig. 8

not shown in the diagram, these being left for the teacher to judge for himself; otherwise it would make the drawing appear unnecessarily complex. The direction of the sun at noon should be noted, and the points of the compass shown after actually

observing the same. It will, of course, be best for the children to make the plan of their own school, and not the one here given. The model thus made by the children will be found extremely useful in many subsequent exercises.

Problems and Exercises—I.

Example of Working.

Find the area of a floor 6 yds. 2 ft. long and 4 yds. wide.

Length of room	= 6 yds. 2 ft.
,, ,, ,,	= 20 ft.
Width of room	= 4 yds.
,, ,, ,,	= 12 ft.
Area of room	= 20 ft. × 12 ft.
,, ,, ,,	= 240 sq. ft.
,, ,, ,,	= 26 SQ. YDS. 6 SQ. FT.

1. A garden is 60 ft. long and 36 ft. wide; find the area in square yards and feet.

2. Find the area of a full size football field, *i.e.*, 120 yds. long and 80 yds. wide.

3. What is the area of a tennis court if it is 36 ft. wide and 80 ft. long?

4. A flag is 7 yds. 2 ft. long and 4 yds. 1 ft. wide; what is its area in square feet and in square yards?

5. A page of an ordinary newspaper is 2 ft. 6 in. long and 2 ft. wide. Find the number of square inches it contains. Also find the area of 8 pages in yards and feet.

6. In a book there are 120 pages; each page is 4 in. × 6 in. Find the number of square inches all the pages would cover, and the number of square yards, square feet, and square inches in the book.

7. In Fig. 8 find the perimeter of the school in yards and feet, the perimeter of the hall, and of each classroom and porch.

8. Find the area of the hall and classrooms in Fig. 8, also of the porches, making due allowance for thickness of walls. Find the total floor space in the school and the total area covered by the building.

9. Find the perimeter of the picture and of the frame in Fig. 7. Calculate the number of square feet in the frame and the amount of moulding wasted in making it.

10. If a table top is $5\frac{1}{4}$ sq. ft. in area, how many square inches does it contain?

11. Calculate the number of square inches in a tablecloth whose area is 21 sq. yds. 6 sq. ft. 108 sq. in.

12. What is the area in yards, feet, and inches of a carpet containing 12,989 sq. in.?

13. Find the area of a floor which contains 934763 sq. in.

14. If a tile is 6 in. square, how many will be required to cover a floor 8 ft. long and 6 ft. 6 in. wide ?

15. I build a wall 3 ft. high and 8 yds. long ; it takes 6 bricks to make a square foot of wall. How many shall I want to build it ?

16. There are 24 sheets of paper to a quire and 20 quires to a ream. If one sheet 20 in. × 12 in. weighs 1 oz., what is the weight of a ream, and what is the area this quantity would cover ?

3. LINEAR AND SQUARE MEASURE.

(1) **Rod, Pole, or Perch.** Tell children to draw a line $5\frac{1}{2}$ yds.

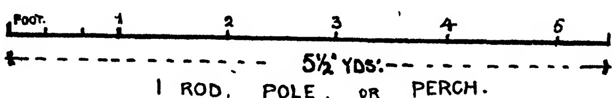


FIG. 9

long to a scale of 1 in. to 1 yd. Mark off into yards and feet, etc., as shown in Fig. 9 ; this is, of course, only a *representation* of a rod or pole. Let children measure off in the classroom a line $5\frac{1}{2}$ yds. long ; this is called a linear rod, polè, or perch.

Next, let children construct a square on a $5\frac{1}{2}$ in. line, and rule it up into representative square yards, etc., as in Fig. 10. Count up the number of square yards in the whole square, thus :—

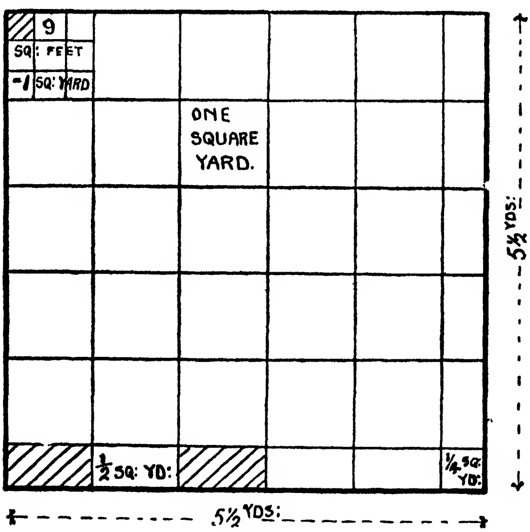


FIG. 10

25 whole squares	=	25	sq. yds.
10 half ,,	=	5	,,
1 quarter square	=	$\frac{1}{4}$	,,
Total	=	$30\frac{1}{4}$	,,

Work arithmetically by multiplying $5\frac{1}{2}$ by $5\frac{1}{2}$, thus :—

$$\begin{array}{rl} 5\frac{1}{2} & \\ 5\frac{1}{2} & \\ \hline 27\frac{1}{2} & = 5 \text{ times } 5\frac{1}{2} \\ 2\frac{3}{4} & = \frac{1}{2} \quad ,, \quad 5\frac{1}{2} \\ \hline 30\frac{1}{4} & = 5\frac{1}{2} \quad ,, \quad 5\frac{1}{2} \end{array}$$

$5\frac{1}{2}$ yds. = 1 rod, pole, or perch (linear)
$30\frac{1}{4}$ sq. yds. = 1 pole (square).

Give some exercises, similar to those in Part III, Section 12 (1) and (2), in graphic representation and simple reduction, both upward and downward.

Tell children to draw to a scale of ·3 in. to 1 yd. (or ·1 in. to 1 ft.) an oblong representing a plot of ground 12 yds. 2 ft. long and 9 yds. 1 ft. wide, and cut this out. Rule up into square yards as shown in Fig. 11, and mark out the square poles (note that there must be $30\frac{1}{4}$ sq. yds. to make a square pole), shading each one in a different way. Now, then, for the remainder. It will be seen that there is not sufficient area left to make another square pole. On counting up, we find 16 whole square yards, two half

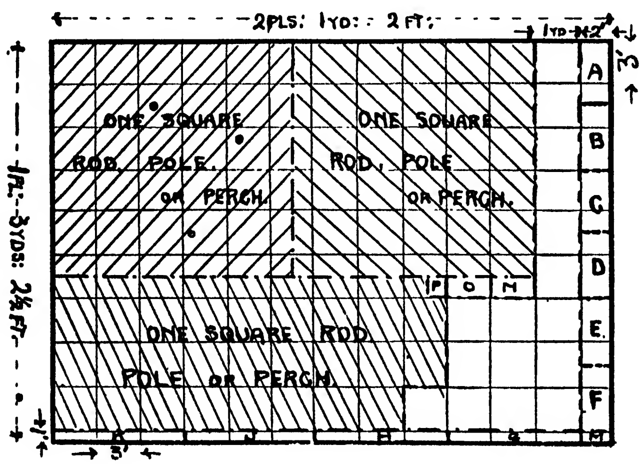


FIG. 11

square yards (O and N), one quarter square yard (P), and the oblongs on the right and along the bottom. A glance at those on the right will show that each one is 6 sq. ft. in area ; then $1\frac{1}{2}$ of these oblongs will be equivalent to 1 sq. yd. ; mark them up as indicated, giving a total (A to F) of 6 sq. yds. Those along the bottom are each 3 sq. ft. in area, then G, H, J, K, are each equal

to 1 sq. yd. in area. The small rectangle (M) at the bottom right-hand corner is 2 ft. long and 1 ft. wide, covering an area of 2 sq. ft.

Totalling up the parts, we find :—

3 sq. poles		
	16 whole sq. yds.	
	6 (A to F) „	
	4 (G to K) „	
	1¼ (N,O,P) „	
		2 sq. ft. (M)
3 sq. pls.	27¼ sq. yds.	2 sq. ft.
= 3 sq. pls.	27 sq. yds.	4¼ sq. ft.
= 3 SQ. PLS.	27 SQ. YDS.	4 SQ. FT. 36 SQ. IN.

Having worked this graphically, it should then be worked out arithmetically, thus :—

Length of plot = 38 ft.
Width „ = 28 ft.
Area „ = 38 × 28.
= 1,064 sq. ft.

9) 1,064 sq. ft.
30¼) 118 sq. yds. + 2 sq. ft.
3 sq. pls. + 27¼ sq. yds. + 2 sq. ft.
= 3 SQ. PLS. 27 SQ. YDS. 4¼ SQ. FT.

Compare these two results and note the different stages in each case. It will be well to reduce the length and width of the plot to feet, and work the scale drawing in yards and feet.

In order to make these two measures more real and practicable the children should certainly be taken into the playground or garden, and there actually measure and mark out with marking pegs a linear pole and then the square pole. The yard measure made earlier will do very well for this, or a piece of string a rod or pole in length will serve admirably.

(2) Furlong, Mile. In the same way deal with the remainder of Linear Measure as was done in the case of yards, feet, and

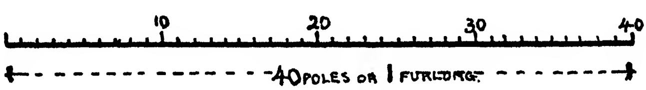


FIG. 12

inches. In Fig. 12 a line 4 in. long represents a furlong or 40 poles, one-tenth of an inch to a pole. Cut off a piece of string eight times the length of this, and this length will represent 1 mile.

(3) **Rood, Acre, Square Mile.** Rule up an oblong 5 in. × 8 in. into small squares, each side of which is ½ in., and let the whole oblong (Fig. 12*a*) represent an acre. In this case, then, one square will represent a square pole and 40 of them 1 rood (cf. "rod" in linear measure). Thus will be represented the comparative area of a pole, a rood, and an acre. Tell the children that 640 of these acres make 1 square mile.

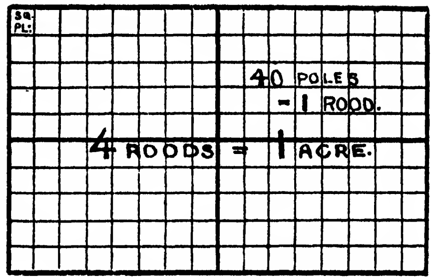


FIG. 12*a*

(4) **Comparative Areas and Population.**

(i) As an exercise in graphic representation, we may take the area and population of the British Isles and its component parts, using round numbers for the sake of simplicity.

Area. Draw and cut an oblong 5 in. × 6 in., and divide each side into half inch parts. Rule up into half inch squares, and let each square represent 1,000 square miles [Fig. 13 (*a*)]. This

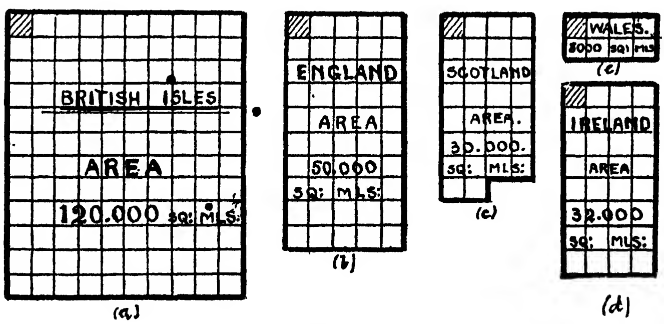


FIG. 13

oblong will then represent the area of the British Isles (ignoring the smaller islands). Cut out another oblong representing the area of England—50,000 sq. miles (*b*), one for Scotland (*c*), one for Ireland (*d*), and another for Wales (*e*).

Population.—This should be worked out to a scale of ¼ sq. in. to a million people. Rule and cut out an oblong 3 in. × 3½ in., and divide each side into half inches and the oblong into half inch squares. This oblong, containing 42 quarter square inches, represents the population of the British Isles. On this basis, then, 32 such squares will represent the population of England, 4½ squares for Scotland, 4 for Ireland, and 1½ for Wales.

These oblongs in Fig. 14 should be of different colour from those in Fig. 13, and should be arranged in column form, the "areas"

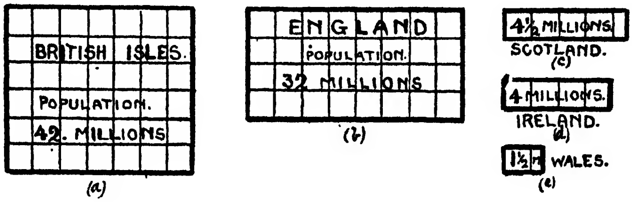


FIG. 14

down one side of book or paper, and the "populations" side by side down the other, giving opportunities for comparison and for observing the density of population.

(ii) As another example, take the comparative areas and

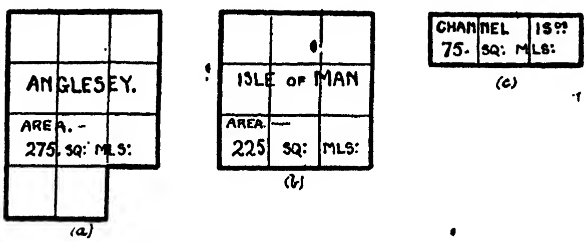


FIG. 15

populations of the Isle of Man, Anglesey, and the Channel Islands. Take, as the basis in Fig. 15 (areas), a half inch square as representing 25 sq. miles. Then Anglesey (*a*) will take 11 of these, Isle of Man (*b*) 9, and the Channel Islands (*c*) 3. The basis of the population representation is the square inch for 100,000, as in Fig. 16 (*c*); this divided into 100 squares of ·1 in. side gives 50 for Isle of Man (*b*) and 60 for Anglesey (*a*), or every 1,000 of the population is represented by one-hundredth of a square inch.

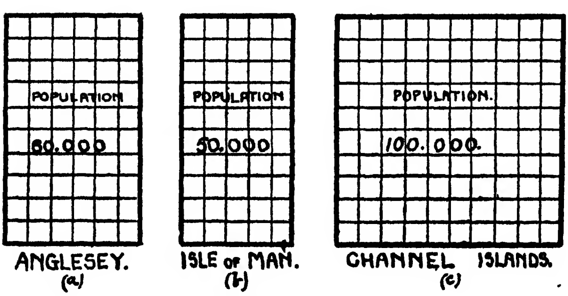


FIG. 16

Problems and Exercises—II.

1. There are 6 telegraph poles in High Street, each one being 1 pole 2 yds. long. If placed upon each other, how high would they reach ?

2. A paving stone is 1 yd. wide and 2 yds. long. The pavement is formed by laying them side by side ; if the street in which I live is a quarter of a mile long, how many paving stones are there ?

3. The sides of a field are 65 yds., 23 yds., 72 yds., and 98 yds. long respectively. What is the distance round the field ? Make a sketch.

4. If a pin measures 1 in. in length and there are 8 rows with 22 in a row, how long would they reach if placed end to end ?

5. Find the area of a garden which is 64 yds. long and 31 yds. wide. Prove your answer by drawing a diagram, scale ·1 in. to 1 yd.

6. The distance round a square is 56 in. How long is each side and what is the area ? Illustrate your answer with a drawing.

7. A piece of rope 36 yds. long goes round the roller for a well 54 times. How far is it round the roller ?

8. If the distance round an equilateral triangle is 48 yds., how many feet long is each side ?

9. A field having an area of 1 acre 2 roods is divided into 8 equal plots. How many square yards are there in each plot ?

10. A cart wheel is $3\frac{1}{2}$ yds. round it ; how many times will it turn round in going 7 miles ?

11. How many plots of ground containing 25 sq. yds. each could be cut out of 2 roods 20 poles ?

12. If a house and garden occupy 360 sq. yds., how many can I put on a piece of ground 60 yds. wide and 72 yds. long ? Illustrate your answer with a sketch.

4. MULTIPLICATION AND DIVISION OF FRACTIONS.

(1) Cancelling.

To divide top and bottom numbers of a fraction by the same number does not alter its value.

(i) It has already been shown that $\frac{5}{10} = \frac{1}{2}$, and that $\frac{2}{4} = \frac{4}{8} = \frac{8}{16} = \frac{1}{2}$; now we will examine other fractions, *e.g.*, the twelfth. Cut out an oblong 3 in. × 4 in. in thin cardboard and rule up into square inches (Fig. 17). Find the area, and note that 1 sq. in. is one twelfth of the whole oblong.

Tell children to cut out from other similar oblongs $\frac{2}{12}$, $\frac{3}{12}$, $\frac{4}{12}$, $\frac{6}{12}$, $\frac{8}{12}$, $\frac{9}{12}$, $\frac{10}{12}$. Test each one of these separately on the whole oblong, and show that $\frac{2}{12}$ covers the same area as $\frac{1}{6}$, that is, by dividing top and bottom by *two* the value remains the same ; again, $\frac{3}{12}$ covers the same area as $\frac{1}{4}$ of the whole, so that by dividing top and bottom by *three* the value remains unaltered. Again, $\frac{6}{12} = \frac{3}{6} = \frac{1}{2}$. It will be obvious that $\frac{6}{12}$ is equal to a half

on superposing on the original unit a part equal to $\frac{6}{12}$; and on comparison with $\frac{1}{6}$ of the oblong, $\frac{6}{12}$ will be seen to contain three of such sixths. Hence it will be seen that sometimes it is possible to divide the top and bottom by more than one number.

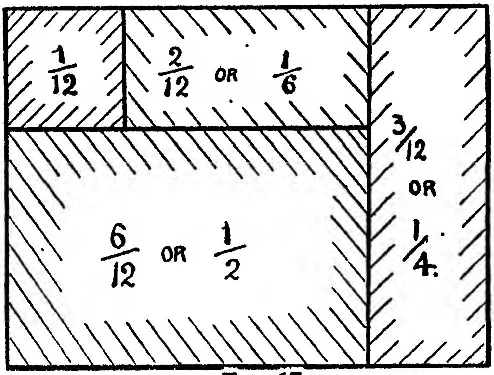


FIG. 17

In the same way illustrate with a line 6 in. long (Fig. 18).

Divide up into halves, quarters, etc., and show that $\frac{8}{12} = \frac{2}{3}$; $\frac{9}{12} = \frac{3}{4}$; $\frac{10}{12} = \frac{5}{6}$.

(ii) Have an oblong of thin cardboard 5 in. × 4 in. ruled up into square inches, as in Fig. 19, and show equivalent fractions to $\frac{2}{20}$, $\frac{4}{20}$, $\frac{6}{20}$, $\frac{8}{20}$, etc. Verify the same by dividing a line 5

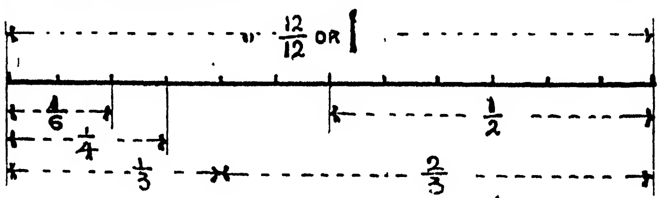


FIG. 18

in. long into twenty equal parts, and marking off the required number of twentieths.

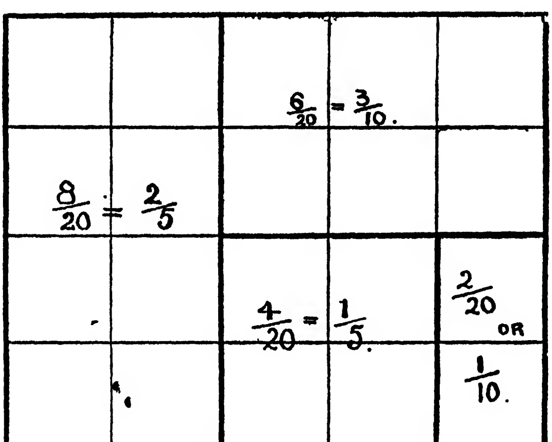


FIG. 19

As an exercise, tell boys to reduce $\frac{12}{16}$ to its *lowest terms*, first of all by diagram and then by arithmetic. Dividing top and bottom by 2 (which is the same as dividing $\frac{12}{16}$ by $\frac{2}{2}$), we get $\frac{6}{8}$; and again by dividing this by $\frac{2}{2}$ we get $\frac{3}{4}$, which is in its lowest terms.

Further exercises should now be given to be worked directly by division.

(2) Multiplication of Fractions.

(i) *To find $\frac{1}{2}$ of $\frac{3}{4}$.* Rule a line 4 in. long (Fig. 20, AB), and divide into quarters. Mark off three quarters (AC), and find a

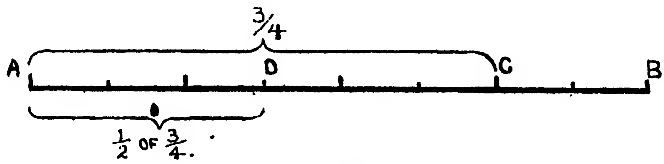


FIG. 20

half of this, which is AD or $1\frac{1}{2}$ in. long. This, then, is $1\frac{1}{2}$ in. of the total length (4 in.), and reducing each to half inches, we get $\frac{3 \text{ half inches}}{8 \text{ half inches}}$ or $\frac{3}{8}$ of whole line.

Take a 4-in. square of paper and fold into oblong quarters (Fig. 21). Fold again down the middle and thus find $\frac{1}{2}$ of the three quarters. Cut this much out, and note that it is of the same area as $\frac{3}{8}$ of the original unit; or expressed in figures—

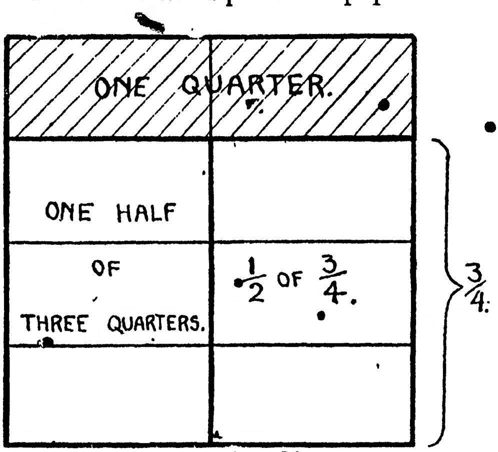


FIG. 21

$$\frac{1}{2} \text{ of } \frac{3}{4} = \frac{1}{2} \times \frac{3}{4}$$
$$= \frac{3}{8}.$$

(ii) *Multiply* $\frac{3}{5}$ by $\frac{5}{8}$. Again work out in line 4 in. long, divided into 8 equal parts (Fig. 22, AB), representing the unit. AC $= \frac{5}{8}$, and of this we require $\frac{3}{5}$, or AD. Now, AD is $\frac{3}{8}$ of the unit line, hence $\frac{3}{5}$ of $\frac{5}{8} = \frac{3}{8}$.

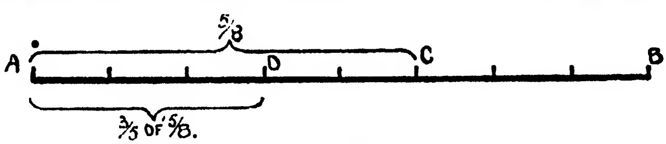


FIG. 22

$$\frac{3}{5} \times \frac{5}{8}$$
$$= \frac{15}{40}$$
$$= \frac{3}{8} \text{ Ans.}$$

$$\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{8}$$
$$= \frac{3}{8} \text{ Ans.}$$

In the working of this arithmetically, it should be pointed out that the "cancelling" may be done in the first line before the multiplication, in order to shorten the working, as is seen on the right-hand side.

Repeat the problem with an oblong of cardboard 2 in. × 4 in., as shown in Fig. 23, giving it an interesting form by saying that "Tom had $\frac{5}{8}$ of a sovereign, and he gave away $\frac{3}{5}$ of what he had to Jack; find how much Jack received." The whole oblong represents the sovereign and the unshaded portion what Jack had, while the part of this given away is indicated by the three squares marked off.

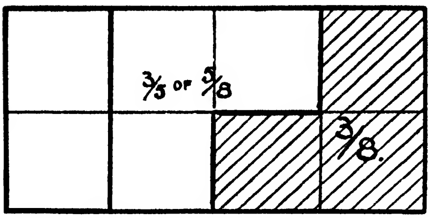


FIG. 23

(iii) *Find* $1\frac{1}{4}$ *times* $2\frac{1}{3}$. Begin with a line 7 in. long (Fig. 24, AB), and mark off into units 3 in. long. Draw also a line (CD), which is a quarter of AB, *i.e.*, $\frac{1}{4}$ of $2\frac{1}{3}$. These two lines, when

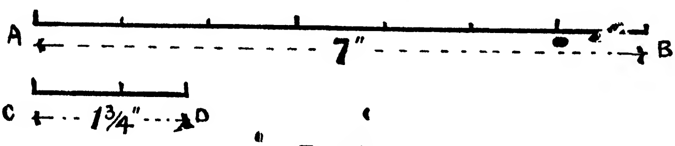


FIG. 24

joined together, will be one and a quarter times AB, or $1\frac{1}{4}$ times $2\frac{1}{3}$. Draw this line and find its length ($8\frac{3}{4}$ in.). Now, as 3 in. represents the unit, this $8\frac{3}{4}$ in. gives 2 units and $2\frac{3}{4}$ in. of another. Reducing this latter quantity to quarter inches, it gives us 11, and since 12 quarter inches make 3 in., it follows that—

$$\frac{2\frac{3}{4}\text{ inches}}{3\text{ inches}} = \frac{11\text{ quarter inches}}{12\text{ quarter inches}} = \frac{11}{12},$$

so that with the two units the result is $2\frac{11}{12}$.

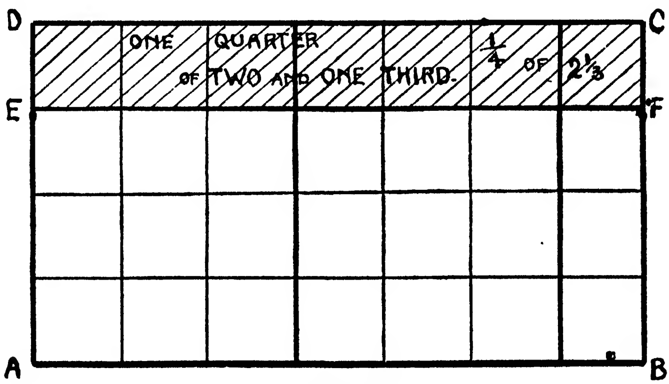


FIG. 25

$1\frac{1}{4} \times 2\frac{1}{3}$
$= \frac{5}{4} \times \frac{7}{3}$
$= \frac{35}{12}$
$= 2\frac{11}{12}$ Ans.

Proceed now to work out arithmetically, as in previous examples. When this is done, repeat the same example, only in area as indicated in Fig. 25.

Build up $2\frac{1}{3}$ units in thin cardboard, each unit containing 12 sq. in., as in Fig. 25, ABCD. Next, mark out one quarter of this (CDEF). This oblong, added to the original, will represent $1\frac{1}{4}$ times $2\frac{1}{3}$. Do this in paper, and note the result. There are the $2\frac{1}{3}$ units + ($\frac{1}{4}$ of $2\frac{1}{3}$, which is equal to $\frac{7}{12}$ of another unit), and that $= 2\frac{1}{3} + \frac{7}{12} = 2\frac{4}{12} + \frac{7}{12} = 2\frac{11}{12}$.

> *Multiplication of Fractions.*
>
> Multiply all the top line figures together for a new top line, and all the bottom line figures for a new bottom line.

(3) Division of Fractions.

(i) *Divide $\frac{5}{8}$ by $\frac{1}{4}$.* It should be noted that in division of fractions, just as in division of whole numbers, the problem is to find the quantity the divisor has to be multiplied by in order to produce the dividend: thus, when 24 is divided by 6, the object is to discover what number 6 must be multiplied by in order to make 24. So with the example above, our object is to discover the fraction by which we must multiply $\frac{1}{4}$ in order to make $\frac{5}{8}$.

Tell the children to draw three lines (Fig. 26) :—

AB (4 in. long) representing the unit,
CD ,, $\frac{5}{8}$,, , and
EF ,, $\frac{1}{4}$,,

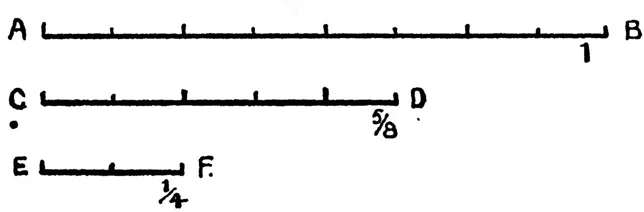


Fig. 26

Since $\frac{5}{8}$ has to be divided by $\frac{1}{4}$, we must see how many times the latter is contained in the former, that is, find by what we must multiply $\frac{1}{4}$ to make $\frac{5}{8}$. On measuring with a strip of paper, it will be seen that CD contains EF $2\frac{1}{2}$ times.

Take the same example in area. Tell the children to rule up a 4 in. square of cardboard into eighths, as in Fig. 27. Cut out $\frac{5}{8}$

(the shaded portion), and arrange two spare eighths to form *a* $\frac{1}{4}$ (the divisor). Try this latter on the $\frac{5}{8}$ piece, and show that it is contained $2\frac{1}{2}$ times in it.

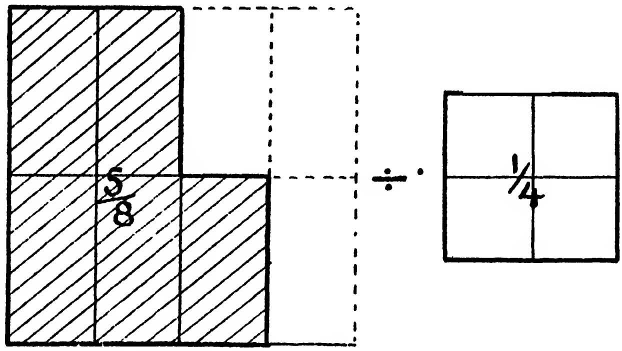


FIG. 27

$\frac{1}{4} \times 2\frac{1}{2}$
$= \frac{1}{4} \times \frac{5}{2}$
$= \frac{5}{8}$.

Prove this by multiplying the divisor by the answer obtained, and note that it is the same as the dividend in the sum.

Reverse this problem, and divide $\frac{1}{4}$ by $\frac{5}{8}$. Again we have to discover what we must multiply $\frac{5}{8}$ by in order to produce $\frac{1}{4}$. It will be seen that AB is $\frac{2}{5}$ of CD, and in order to produce $\frac{2}{5}$ we must find *something* to multiply $\frac{1}{4}$ by. If we invert $\frac{2}{5}$ and multiply the $\frac{5}{2}$ so obtained by $\frac{1}{4}$, we get $\frac{5}{8}$, so that if we invert $\frac{5}{8}$ ($\frac{8}{5}$) and multiply it by $\frac{1}{4}$ we get $\frac{8}{20}$ or $\frac{2}{5}$. Thus to find what we must multiply $\frac{5}{8}$ by in order to produce $\frac{1}{4}$, we invert the dividing fraction.

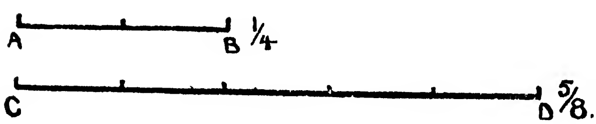


FIG. 28

In the second case (Fig. 29), we still have to find what fraction we must multiply $\frac{5}{8}$ by in order to produce $\frac{1}{4}$. Try the same rule found above, namely, by inverting the divisor and multiplying. Hence $\frac{1}{4} \div \frac{5}{8}$ is the same as multiplying $\frac{1}{4}$ by $\frac{8}{5}$ (that is, $\frac{5}{8}$ inverted). Prove that this is correct by

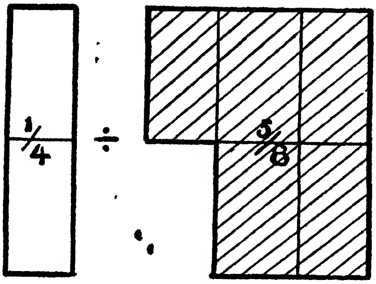


FIG. 29

multiplying $\frac{5}{8}$ by $\frac{2}{5}$. This gives us $\frac{1}{4}$, or the fraction we set out to discover.

$$\begin{aligned}\tfrac{1}{4} \div \tfrac{5}{8} &= \tfrac{1}{\cancel{4}} \times \tfrac{\overset{2}{\cancel{8}}}{5}\\ &= \tfrac{2}{5} \text{ Ans.}\end{aligned}$$

(ii) *Divide* $2\frac{1}{4}$ *by* $\frac{1}{2}$. Tell the children to cut out three 2 in. squares of paper and fold into quarters, cutting one and arranging as in Fig. 30. We have to see how many times $\frac{1}{2}$ is contained in $2\frac{1}{4}$, or to find the fraction we must multiply $\frac{1}{2}$ by in order to produce

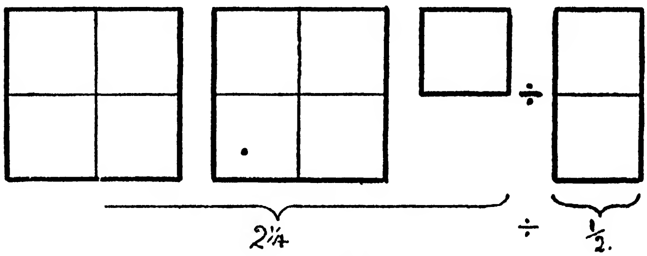


FIG. 30

$2\frac{1}{4}$. It is clear that the 2 squares composing the oblong half are contained in the 9 squares composing the $2\frac{1}{4}$ units, $4\frac{1}{2}$ times. Hence we must multiply $\frac{1}{2}$ by $4\frac{1}{2}$ in order to make $2\frac{1}{4}$.

$$\begin{array}{lll}
\tfrac{1}{2} \times 4\tfrac{1}{2} & 2\tfrac{1}{4} \div \tfrac{1}{2} & \tfrac{1}{2} \div 2\tfrac{1}{4} \\
= \tfrac{1}{2} \times \tfrac{9}{2} & = \tfrac{9}{4} \div \tfrac{1}{2} & = \tfrac{1}{2} \div \tfrac{9}{4} \\
= \tfrac{9}{4} & = \tfrac{9}{\underset{2}{\cancel{4}}} \times \tfrac{\cancel{2}}{1} & = \tfrac{1}{\cancel{2}} \times \tfrac{\overset{2}{\cancel{4}}}{9} \\
= 2\tfrac{1}{4} \text{ Ans.} & = 4\tfrac{1}{2} \text{ Ans.} & = \tfrac{2}{9} \text{ Ans.}
\end{array}$$

By examining the working on the left, the result of the practical work is seen to be correct; for we found that $\frac{1}{2}$ was contained in $2\frac{1}{4}$, $4\frac{1}{2}$ times.

Applying the same rule observed in Case (i), we see that the $2\frac{1}{4}$ squares are $4\frac{1}{2}$ times (or $\frac{9}{2}$ times) the oblong half, and in reversing the sum, as is done on the right above, we find that $2\frac{1}{4}$ must be multiplied by $\frac{2}{9}$ (or $4\frac{1}{2}$ inverted) in order to make $\frac{1}{2}$.

Hence, once more the rule, invert the divisor and multiply.

Now assume the answer to the sum given is obtained correctly; let the children verify it graphically by the aid of two lines, taking

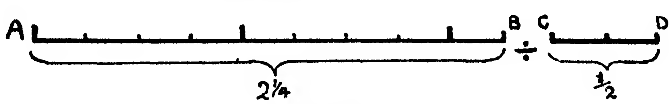


FIG. 31

a line 1 in. long as the unit. Here we have to see how many times the $\frac{1}{2}$ in. line (Fig. 31, CD) is contained in the $2\frac{1}{4}$ in. line (AB).

Use a strip of paper with length of CD marked on the edge, and try this along AB; or use the rulers similarly. CD will be found to measure $4\frac{1}{2}$ times along AB.

(iii) *Divide* $3\frac{1}{4}$ *by* $1\frac{1}{2}$. Tell the children to draw two lines, 1 in. to the unit, representing $3\frac{1}{4} \div 1\frac{1}{2}$, as in Fig. 32. Let them discover for themselves, by measurement, that the divisor is contained in

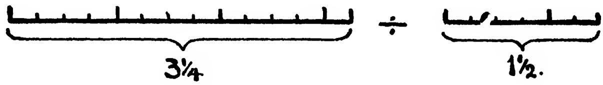


FIG. 32

the dividend twice, and a quarter of an inch to spare; but as there are 6 quarter inches in the divisor, this gives us an answer to the sum as $2\frac{1}{6}$.

Working.

$$3\tfrac{1}{4} \div 1\tfrac{1}{2} = \tfrac{13}{4} \div \tfrac{3}{2}$$
$$= \tfrac{13}{\cancel{4}_2} \times \tfrac{\cancel{2}}{3}$$
$$= \tfrac{13}{6}$$
$$= 2\tfrac{1}{6} \text{ Ans.}$$

Proof of Working.

$$1\tfrac{1}{2} \times 2\tfrac{1}{6} = \tfrac{\cancel{3}}{2} \times \tfrac{13}{\cancel{6}_2}$$
$$= \tfrac{13}{4}$$
$$= 3\tfrac{1}{4} \text{ Ans.}$$

Let this sum be verified by working out in area, as shown in Fig. 33. It will be seen that the $3\frac{1}{4}$ contains 13 small squares

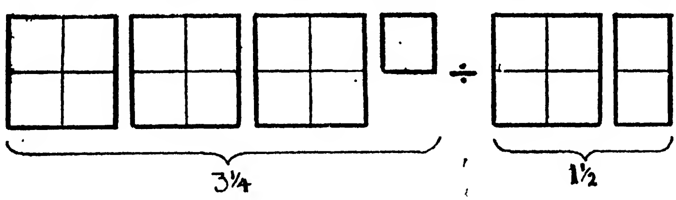


FIG. 33

and the $1\frac{1}{2}$ contains 6. Six is contained in 13 twice and one-sixth remaining, or $2\frac{1}{6}$ times. The working out by arithmetic is shown on the left above.

5. HANDWORK EXERCISES.

Envelope Case (Fig. 34). This model should be made in thin cardboard, but it may also be made in stiff paper. Draw along the dotted lines of the diagram and cut the firm ones; the two shaded portions are each 1 sq. in. in area, as will be observed. Half cut along the dotted lines and bend away from the cut. Fold up into position and fasten together with Seccotine: A should be fastened to the inside of C, and B to the inside of D first, then E and F to the outside of C and D respectively. As a further

exercise in cutting of curves, the case may be cut as shown in the dotted lines of the general view.

Sweet Box (Fig. 35). Rule up with the aid of set-squares the development as shown, and draw in the curved tabs for slipping

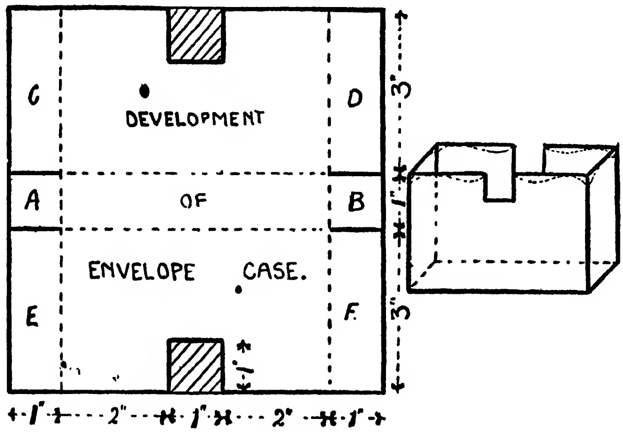


Fig. 34

into the slits A and B. Half-cut along the dotted lines, cut out the shaded part, and fold up into position. Mark the position of the slits A and B from the tabs while folded up ready for sticking the flange (*f*) to the side F. Fold inwards the square flaps, and

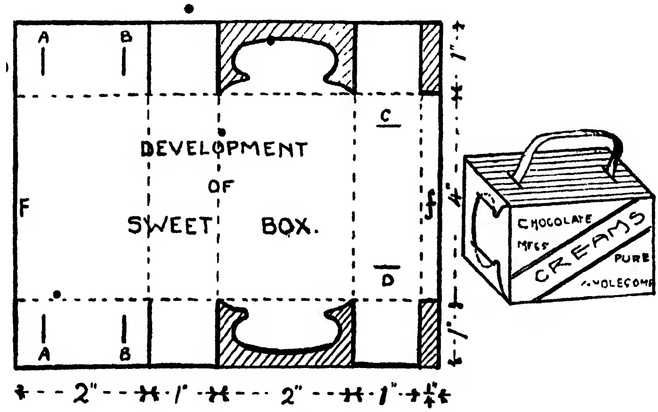


Fig. 35

insert the tabs in their respective slits. The slits C and D are for the insertion of narrow tape or bébé ribbon; fasten these on the inside of the box, and over each end fasten a piece of paper to keep it from slipping out of place.

Problems and Exercises—III.

1. Show by a diagram in two ways that $\frac{16}{24}$ is the same in value as $\frac{4}{6}$ and is equal to $\frac{2}{3}$.
2. Reduce $\frac{9}{12}$, $\frac{4}{8}$, $\frac{6}{10}$, $\frac{12}{16}$, $\frac{16}{20}$, and $\frac{24}{32}$ to their lowest terms.
3. Show by a diagram that twice 2·5 is equivalent to twice $2 \times 2\frac{1}{2}$.
4. Find how much of an orange Tom gets if Jack gives him $\frac{1}{4}$ of what he has, when Jack has $\frac{2}{3}$ of an orange. Illustrate your working by a line.
5. If I have $5\frac{2}{3}$ shillings and my brother has $3\frac{1}{2}$ times as much, how much has he ? How many pence would this be ?
6. I have $10\frac{2}{3}$ shillings in my pocket ; if I divide it equally among four boys, how much will each get ? Show this is correct by drawing a line 1 in. for every shilling.
7. A table top is $\frac{5}{6}$ of $2\frac{1}{2}$ yds. long and $\frac{3}{4}$ of $1\frac{1}{2}$ yds. wide ; find its length and width in feet and inches.
8. A newspaper when opened out covers an area of 12 sq. yds. How many of these newspapers would it take to cover a floor which is 15 yds. long and 12 yds. wide ?
9. Find the area of each of the parts of the Envelope Case (Fig. 34), and so find the total area. Compare this with the development in the same figure, and find the difference.
10. In Fig. 34, find how much A is of E and how much E is of the side. Divide the area of the bottom by that of an end ; prove by diagram.
11. Calculate the area of the cardboard used in the development for the Sweet Case (Fig. 35), the area of the flange (*f*), and the top and sides.
12. What is the longest perimeter of the Sweet Box and what is the shortest ? Find the length of the other perimeter.

6. DECIMALS—THE SIMPLE RULES.

(1) The Hundredth.

Following on the first decimal place, the hundredth logically follows. Tell children to draw a line 10 in. long and divide it into tenths. Divide the first inch into tenths again (Fig. 36).

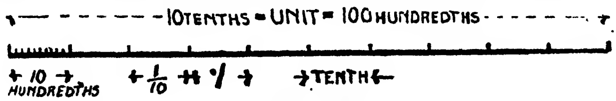


FIG. 36

The whole line which represents a unit is now divided into ten tenths ; the first tenth is also divided into ten tenths ; and since there are 100 of these in the whole line (10 tenths, with 10 divisions in each tenth = 100 hundredths), each one must represent $\frac{1}{100}$ of the whole line. Work out fractionally one tenth of one tenth = $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$.

Next, let children rule up a 5 in. square into ten strips $\frac{1}{2}$ in. wide—10 tenth strips. Each one of these strips represents ·1 of the square. Divide the opposite sides into tenths, and rule across the strips (Fig. 37). Now cut the tenths strips down; take one of these strips and cut off a tenth of it—that is, $\frac{1}{10}$ of $\frac{1}{10}$ = $\frac{1}{100}$ of the whole. Calculate the number of these small squares in the original, and compare with the fractional method for verification.

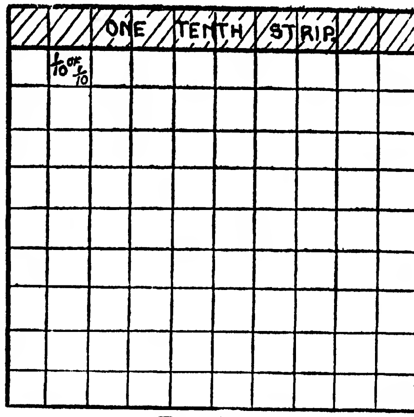


FIG. 37

Refer to Part III (Figs. 2 and 6) and again revise local values of figures. Continuing this method, the natural position for the hundredth in decimals is on the right of the tenth, since the tenth is on the right of the unit, etc. This is the method in use. Tell children to build up 13 hundredths, 47 hundredths, 79 hundredths (Fig. 38), and express them fractionally $\frac{13}{100}$, $\frac{47}{100}$, $\frac{79}{100}$. Let us examine the first group of hundredths—13 of them. There are 10 hundredths in the long strip (the tenth of the whole square) and 3 separate hundredths. Now we know that the 10 hundredths were made one tenth of the square originally, so we can express that as heretofore, namely, ·1; but the 3 hundredths have also to be indicated, and this is done on the right of the tenth, so ·13. This 3, or whatever the figure may be next to the tenth (or first decimal place) on the right, is called the *second decimal place.* In the second example there are 4 tenth strips, and 7 hundredths indicated thus: ·47. This is to represent 47 hundredths or 4 tenths and 7 hundredths. In the third example there are 79 hundredths or 7 tenths (·7) and 9 hundredths.

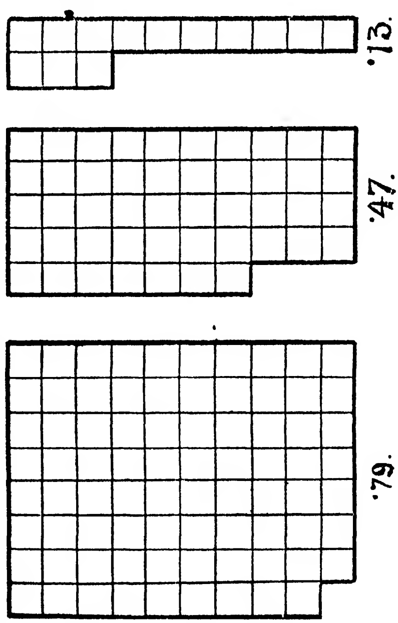


FIG. 38

Suppose we have to express in figures $\frac{6}{100}$ (Fig. 39); we know from our hundredth strips that this $\frac{6}{100}$ is not sufficient to make one tenth, so we place the decimal point and a "nought" after it to indicate that there are no tenths, and then place the 6 after that to represent $\frac{6}{100}$ or ·06. The same applies to $\frac{3}{100}$, $\frac{5}{100}$, etc.; the first decimal place always represents tenths, the second decimal place always represents hundredths.

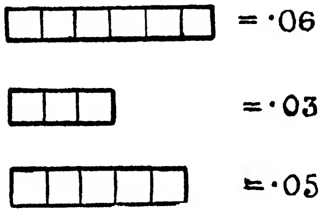


FIG. 39

(2) Addition.

Several examples in addition should be worked out with the strips, before commencing purely figure work. Take examples without any "carrying," first omitting whole numbers, then one involving "carrying" and whole numbers.

Add ·25, ·04, *and* ·47. Build up each line separately in tenth and hundredth pieces (Fig. 40). In adding, take all the hundredth pieces and with them make as many equivalent tenth pieces as possible; in this case there are 16 hundredth pieces—therefore 1 tenth piece and 6 hundredth pieces. Set down the result in correct form, and underneath the tenths figure set 6 for the whole tenth strips. Add these two lines together, and the result is ·76; verify by counting up the hundredths in each strip.

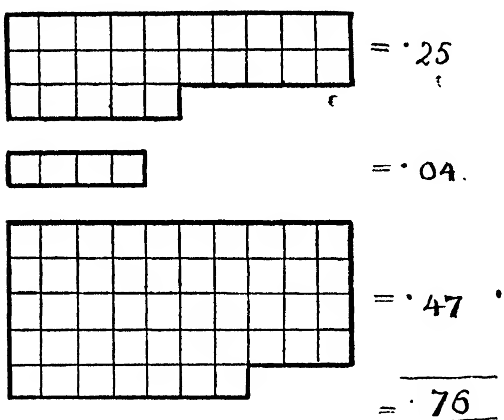


FIG. 40

In the same way deal with the tenths when there are sufficient to make a unit (or units) to add on to the units column, and so on.

$$\begin{array}{l} \cdot16 = 16 \text{ hundredths} \\ \cdot6 \;\; = \;\; 6 \text{ tenths} \\ \hline \cdot76 = 76 \text{ hundredths} \end{array}$$

(3) Subtraction.

(i) *Take* ·32 *from* ·47. Build up the ·47 line in strips and the ·32 also. From the top group (Fig. 41) take a number of the necessary strips equal to those of the bottom group. The remainder will be the result, namely, ·15. Work in figures, as in Fig. 41.

(ii) *From* 1·04 *take* ·57. Tell the children to rule up a 5 in. square into hundredths; also to cut out a piece to represent ·04 (Fig. 42). From this has to be taken ·57; mark off a piece equivalent to 57 hundredths (·57); the remaining portion consists of 4 tenths and 7 hundredths, that is, ·47.

In working out in figures, change the 1 unit into 10 tenths and use one of these tenths by converting it into hundredths and adding on the 4, making 14 hundredths. This leaves 9 tenths, from which 5 have to be taken, leaving 4 in the answer ·47.

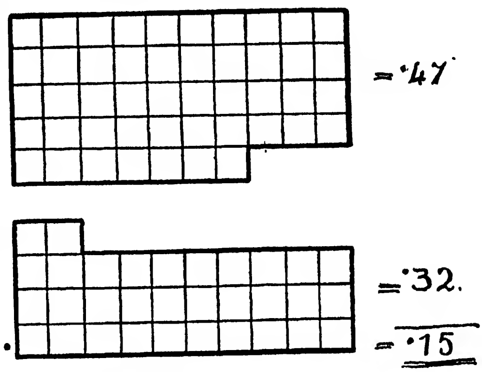


FIG. 41

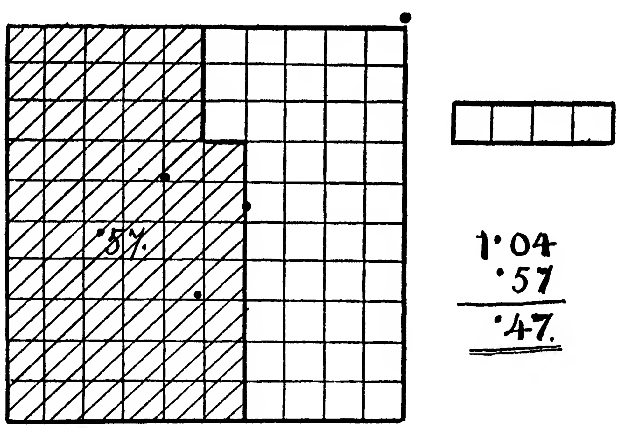


FIG. 42

(4) Multiplication.

(i) *Multiply* 2·54 *by* 3.

Tell children to build up with strips already cut, 2·54 three times. Count up the number of whole units = 6; then the number of tenths = 15, or 1 unit and 5 tenths; and, lastly, the number of odd hundredths = 12, these are equivalent to 1 tenth and 2 hundredths.

2·54	6·0	= whole units
3	1·5	= 15 tenths
	·12	= 12 hundredths
7·62	7·62	

(ii) *Multiply* 2·5 *by* ·3. Have an oblong 6¼ in. × 2½ in. ruled up as in Fig. 43. This represents 2·5 units, and we have to obtain

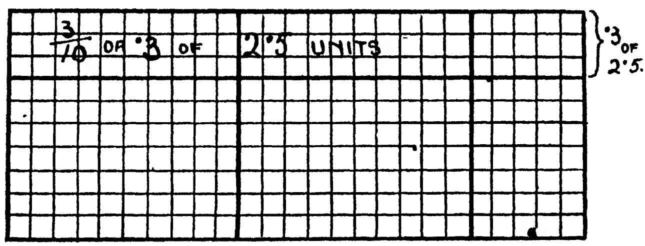


FIG. 43

·3 of 2·5 units. Rule off ·3 of the unit, and shade this portion of the diagram and find the number of whole tenths (6), and the number of tenths that can be made from the 15 hundredths at the end of the figure (1), leaving 5 hundredths over. This, when set down in the form of a sum, gives—

6 tenths	= ·6		2·5
1 ,,	= ·1		·3
5 hundredths	= ·05		·75
	·75		

The answer shows 75 hundredths; prove this by finding the area of the shaded part, taking a hundredth square as the unit.

(iii) *Multiply* 2·3 *by* 2·7. Tell the children to build up with their strips 2·3 units; repeat this, and add ·7 times underneath, as in Fig. 44. Set down, as in former examples, (1) the number of whole units, (2) the number of whole tenths, and (3) the number of hundredths left :—

(1) 4 whole units		= 4·0
(2) 20 ,, tenths	= 2 units	= 2·0
(3) 21 hundredths	= 2 tenths and 1 hundredth	= ·21
		6·21

Verify now by multiplication :—

$$\begin{array}{l} 2{\cdot}3 \\ 2{\cdot}7 \\ \hline 4{\cdot}6 = 2{\cdot}\quad \text{times } 2{\cdot}3 \\ 1{\cdot}61 = {\cdot}7 \quad ,, \quad 2{\cdot}3 \\ \hline 6{\cdot}21 = 2{\cdot}7 \quad ,, \quad 2{\cdot}3 \end{array}$$

Have other examples worked out practically, first either by drawing suitable oblongs or by building up with the square inch, tenths, and hundredths, or cutting out to a different scale, larger whenever possible; then have these verified by multiplication, etc. Care

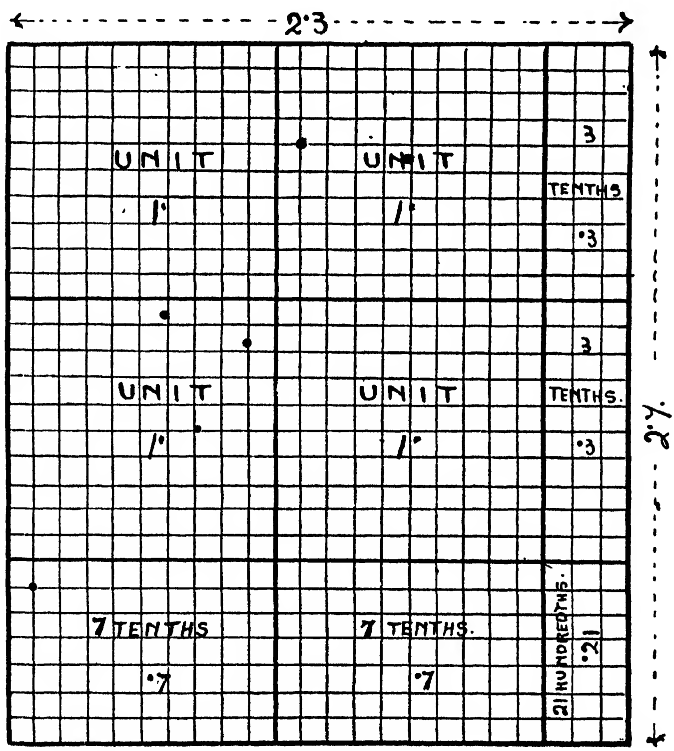


FIG. 44

should be exercised in giving problems that will involve any difficulty not dealt with up to the time; and care in giving exercises which are capable of graphic representation to a large extent.

(5) Division.

(i) *Divide* 2·76 *by* 3. Continuing the process explained in Part III, section 7, have 2·76 units built up and divided into three equal parts (Fig. 45). There are insufficient units to be divided as such by three, so we change the two units into 20 tenths; this and the 7 tenths make 27 tenths, and dividing into three equal parts gives 9 tenths (or ·9) to each third. The 6 hundredths are easily divided into three equal parts, 2 hundredths to each part.

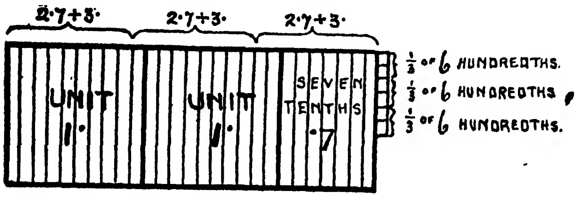


FIG. 45

State in the decomposed form, and afterwards in the usual way :—

$$\frac{3)27 \text{ tenths} + 6 \text{ hundredths}}{9 \quad ,, \quad + 2 \quad ,,} = \frac{3)2{\cdot}76}{{\cdot}92} \text{ Ans.}$$

(ii) *Divide* 1·2 *by* ·8. Tell children to cut out an oblong which will represent 1·2 and another (of a different colour preferably) to represent ·8 (Fig. 46). We have to see how many times ·8 of a unit is contained in 1·2. Superpose the divisor on the dividend, and notice that it is contained one and a half times, or 1·5 times.

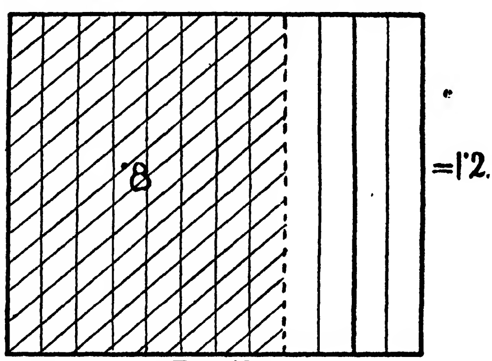


FIG. 46

Again, in dividing ·8 into 1·0 :—

$$\frac{{\cdot}8)1{\cdot}2}{1{\cdot}5} = \frac{8 \text{ tenths})12 \text{ tenths}}{1\tfrac{1}{2} \text{ tenths or } 1{\cdot}5}$$

·8 is contained once and 2 tenths (·2) over; this ·2 and the ·2 still undivided means that ·4 (or 4 tenths) is to be divided by ·8 (or 8 tenths), and this produces the half or ·5.

For the sake of convenience and in order to obviate the necessity of dividing by fractional parts, we usually "make the divisor

into a whole number," so that if we make ·8 into 8·0 we have multiplied it by 10, and in order that the value of the result may be unaltered, we must treat the dividend in the same way. When this is done, the result obtained by dividing 1·2 by ·8 is the same as when dividing 12· by 8·, for 8 into 12 gives us one and $\frac{4}{8}$ (or $\frac{1}{2}$ or ·5) over.

Prove this by a diagram similar to Fig. 45, only using whole numbers instead of decimals.

(iii) *Divide* 5·4 *by* 1·2. Tell children to rule up or build up 5·4 units (Fig. 47), and as far as able to divide the group into smaller

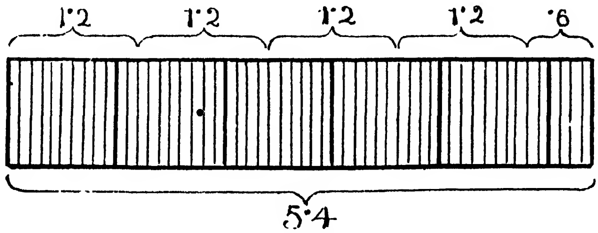


FIG. 47

groups of 1·2 units in each group. This allows of 4 groups and a half (or 4·5). State this in the two ways, as before :—

$$1{\cdot}2\overline{)5{\cdot}4} = 12 \text{ tenths}\overline{)54 \text{ tenths}}$$
$$\underline{4{\cdot}5} \qquad\qquad \underline{4\tfrac{1}{2} \text{ or } 4{\cdot}5.}$$

Or by following the usual rule of making the divisor a whole number, thus :—

$$1{\cdot}2\overline{)5{\cdot}4} = 12{\cdot}\overline{)54{\cdot}}$$
$$\underline{4{\cdot}5} \qquad \underline{4{\cdot}5}$$

Many similar exercises should be given of the same kind, gradually increasing in difficulty as the progress of the class justifies it.

Following on the same principle, the thousandth or third decimal place should follow as a natural sequence, and exercises judiciously chosen to illustrate the use of it.

Problems and Exercises—IV.

1. Draw a line $4\frac{1}{2}$ in. long; express this as a decimal. Find the number of tenths there are in it and the number of hundredths.

2. A line is 265 hundredths of an inch long; measure this off and draw it. Show the number of tenths and units in the line. Express each as a decimal.

3. Express the following as decimals : $\frac{3}{10}$, $\frac{4}{5}$, $\frac{2}{5}$, $\frac{7}{100}$, $\frac{19}{100}$, $\frac{173}{100}$.

4. A boy had 2·35 apples, another 15·04 apples, and a third ·1 of an apple. How much had they altogether ?

5. I walked 8·25 miles, rode 65·32 by train, and 43·07 miles by motor-car. How far did I travel altogether ?

6. The sides of a triangle are 3·42 in., 7·93 in., and 6·08 in. long respectively. What is the distance round it ?

7. I have in my purse 8·13 shillings. I gave away 2·07 shillings to a boy and 3·56 shillings to a girl. How much had I left ?

8. From Walsall to London is 120 miles. If I rode 102·45 miles and walked the rest, how many miles did I walk ?

9. A truck load of coal and the truck together weigh 18·51 tons ; if the truck itself weighs 8·76 tons, how heavy was the coal ?

10. A sack of flour contains enough to make exactly 12 smaller bags, each weighing 24·75 lbs. ; how much flour did the sack contain ?

11. A newspaper contains 8 pages, with 7 columns on each page. Each column has 722·53 words, taking column for column. How many words are there in the whole paper ?

12. Multiply $1\frac{3}{10}$ by $1\frac{1}{2}$, and prove your answer by working the same sum in decimals.

13. I divide a piece of string 24·68 in. long into 4 equal parts ; how long is each part ?

14. An oblong piece of cardboard contains 46·2 sq. in. How many pieces each containing 1·1 sq. in. could I cut from it ?

15. A man had 126·72 pounds, and divided it equally between his six sons ; how many pounds did each one receive ?

7. GEOMETRY—ANGLES.

As linking up previous work on angles, and introductory to the mensuration of the circle, etc., it will be well now to have some practical work on the Circle and Angles.

(1) **Folding and Cutting.** Draw with compasses and cut out in thin paper a circle, opening the compasses exactly 2 in. ; this distance is called the radius.

Fold as shown in Fig. 48, and obtain a semicircle. Have this fully described. Open out and fold again, as in 3 (Fig. 48). Open out again. Rule along the crease marks of 4, forming four right angles. Remind children of the "square corner" or right angle of previous lessons, and tell them that for convenience in reckoning the sizes of angles, they are divided into *degrees*. The right angle, being the standard, is divided into 90 equal parts or degrees, signified by a small ° after the number of degrees. Then ascertain how many there are at the centre of the circle—360°—and tell them that this is the same, whatever the diameter of the circle. Cut out one of the right angles and bisect it by folding, and find the number of degrees in half a right angle. This is called a mitre—*cf.* the corners of a picture frame or some of the models made where the flanges have to fit together to form a right angle.

Bisect this half right angle or mitre, and obtain the number of degrees in this half. Give some exercises in building up various angles: *Acute*, $22\frac{1}{2}°$, $45°$, $67\frac{1}{2}°$; *Obtuse*, $112\frac{1}{2}°$, $135°$, $157\frac{1}{2}°$, etc.

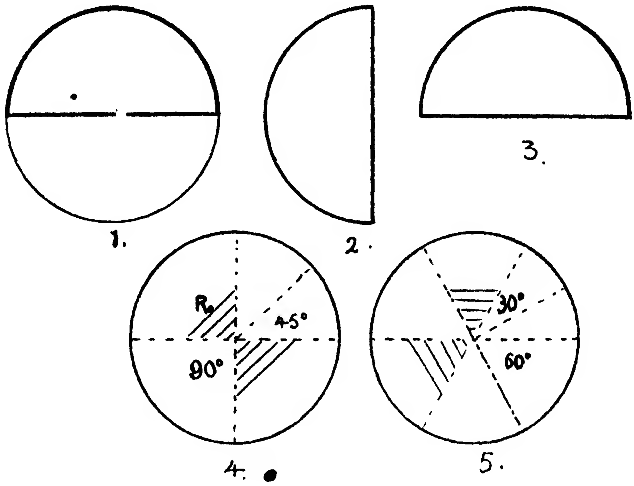


FIG. 48

Cut out another circle and fold it into a semicircle; by trial, divide each half circumference into three equal parts, and fold. Find the number of degrees in each of these parts (Fig. 48, No. 5) by dividing 360° by 6. Then cut out an angle of 60°, bisect and obtain an angle of 30°; bisect an angle of 30°, and obtain one of 15°. Build up various angles with pieces marked with the number of degrees they contain, and paste in exercise books with the name and size of the angle written by it. *Acute Angles*, 30°, 60°, $52\frac{1}{2}°$, $82\frac{1}{2}°$, 75°; *Obtuse Angles*, 120°, 105°, 165°, 150°, etc. Some of the more difficult examples may be omitted, but those given should not prove beyond the capabilities of the children, if the examples are worked *practically*.

(2) **With Compasses.** Draw a line of any length. We wish to find the middle point of the line, or to bisect it. Remind children of halves, quarters, etc., and how we obtained the middle then. Tell children to open their compasses as near to half the line as they can judge, and mark off a point near the middle which shall be the same distance from each end (A and B, Fig. 49).

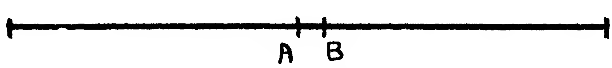


FIG. 49

Now let them mark the middle of this short space, and test for the middle of the line with strips of paper and with compasses.

In order to obviate any difficulty or chance of error, the correct method should be shown, as in Fig. 50. Test again, and prove practically that the line is actually bisected. Tell them to show that any points (A, B, or C) taken on the bisector of the line XY are the same distance from X as from Y. This may be done with their compasses or with strips of paper, or by both methods.

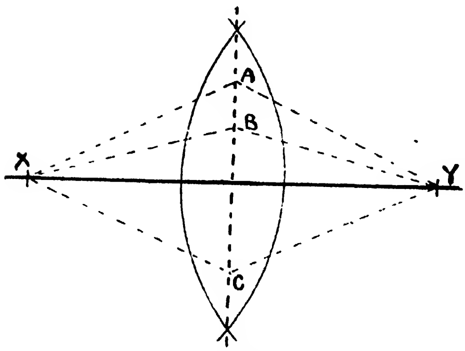


FIG. 50

Tell the children now to draw a circle and its diameter (*cf.* Fig. 48, Nos. 1 and 2). We require now to make an angle of 90° with the compasses (*cf.* Fig. 48, No. 4). It will be seen that having a circle divided into halves, to obtain quarters we must halve the halves, for we want to find the middle of the half circumference and the diameter. Have the folded right angles along side while this drawing is proceeding, and constantly compare. Lead children to use their compasses for this (Fig. 51) instead of folding, as was done previously.

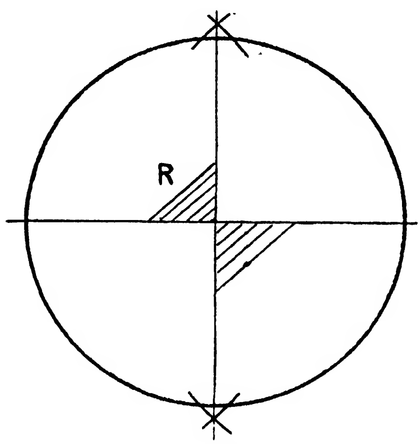


FIG. 51

Superpose a right angle on the divisions obtained in Fig. 51, and test each one. From this proceed to the ordinary method of constructing a right angle geometrically.

Take the half right angle that was folded, and ask children how it was obtained—by bisecting a right angle. Then to obtain it with compasses, the same method is adopted. It is again necessary to find the middle of the arc, and so we bisect it, as we did the line in Fig. 51. Cut the right angle out with scissors and then along the bisector. Superpose each, and compare each angle and see that the arcs of each angle of 45° are the same length.

Tell children to describe a circle, and step off with the same radius six divisions around the circle (Fig. 53) along the circumference. Cut out one of these angles and bisect it by folding. Calculate the number of degrees in the sixth of the circle and

hence in the twelfth. Suppose we wish to make an angle the same as the piece cut out, namely, 60°, the method is practically the same as copying one-sixth of the upper half of Fig. 53, and bisection is the same as in preceding problem.

(3) **Testing Set Squares.** In testing the set squares, either those made by the children themselves or those supplied, the application of much of the foregoing work on angles may be shown. To test for the right angle, place two right angles back to back, standing on a straight edge. If all fit close up to each other, (or "hug" each other) then the right angles may be said to be

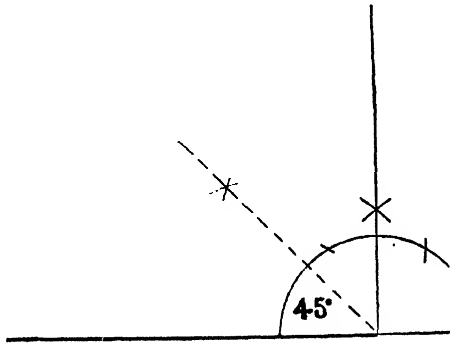


Fig. 52

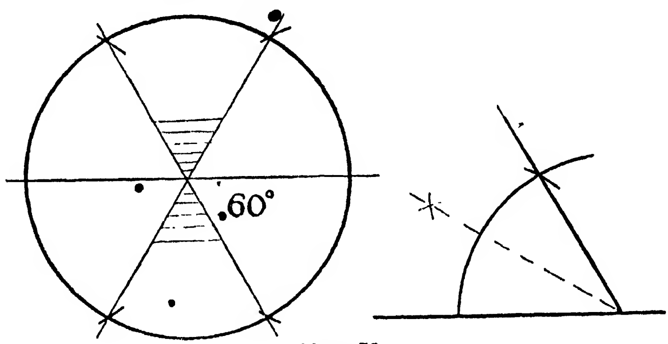


Fig. 53

correct. Another method of testing, and one which applies here, is by drawing a perpendicular and turning the set square over, as in Fig. 54, and drawing another line from the same point on the base line. It will be obvious to the children, after considering Fig. 48, No. 4, that if the right angle of the set square is accurate, these two lines will fall exactly on each other; if not, the lines will appear something like that shown in Fig. 54, No. 2.

The other angles may be tested by a combination of angles, in order to make two right angles, as is illustrated in Fig. 54, Nos. 3 and 4.

(4) **Construction of Simple Scales.** It will be found useful to have handy at times "scales" of various dimensions, and for this purpose they should be constructed in thin cardboard. Have

several pieces cut off 1¼ in. wide and 13 in. long. This will allow of a 12 in. scale on each edge, and ½ in. to spare at each end. The

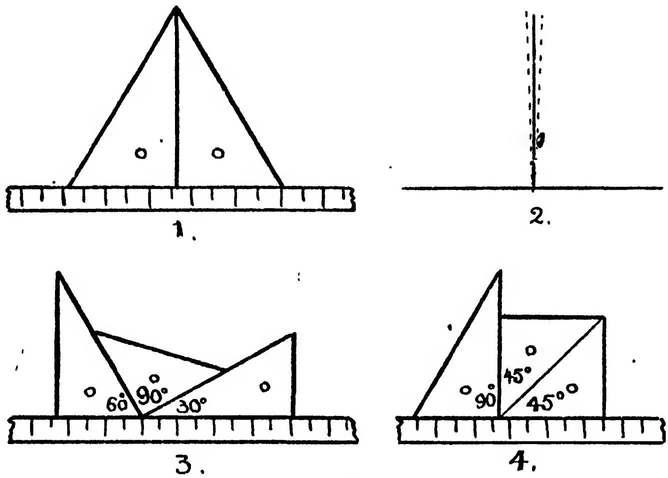


FIG. 54

ruler supplies all wants for full-size scale, but a start may be made with (*a*) ½ in. to 1 in. or half-size scale. This is done on the bottom edge in Fig. 55. Every half inch along the scale represents a real

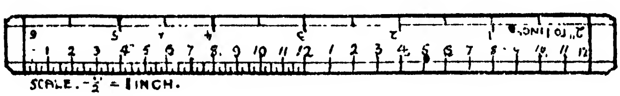


FIG. 55

inch in length. Along the other edge construct a scale of 2 in. to 1 in., or double-size scale (Fig. 55), top edge of diagram.

Various other useful scales should be constructed as circumstances arise—1 in. to 1 ft.; 1 in. to 1 yd., etc.; ½ in. to 1 ft., ¼ in. to 1 ft., $\frac{1}{10}$ in. to 1 ft., etc.

8. HANDWORK EXERCISES.

Roman Letters (Fig. 56). Have several letters similar to those in Fig. 56 worked out to scale; use being made of the scales already constructed. Other objects can be set out and drawn to simple scales—like the school neighbourhood—the distances being stepped off in yards wherever practicable, or in "chains" (22 yds. or 4 rods = 1 chain).

Scale Case (Fig. 57). Make this of good stiff brown paper, and of a size to contain the "scales" that have been constructed. A ruler placed inside while the sticking together is in progress will

considerably facilitate the handling of the work. The bottom should be fixed in the same way as was done with the Envelope

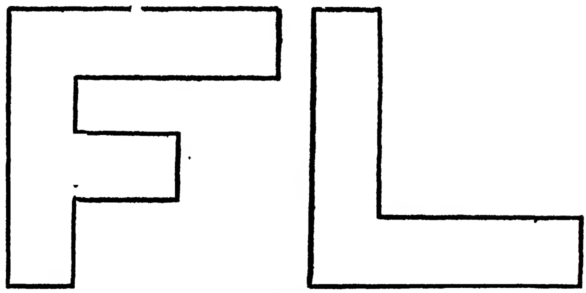


FIG. 56

Case (Fig. 34). The thumb bits at the top can be drawn with compasses, and cut out either with the scissors or point of knife.

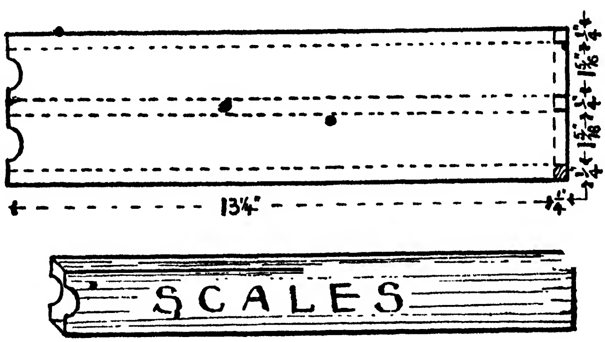


FIG. 57

Problems and Exercises—V.

1. Draw an angle of 90° with your set square, and bisect it; also trisect it.

2. Make an angle of 30° and another twice as great.

3. Draw any triangle and bisect all its angles.

4. Draw a circle, and in it place an equilateral triangle touching the circumference.

5. Draw a line 3·75 in. long and bisect it; bisect each of the parts afterwards.

6. Make an obtuse angle of 120° as nearly as you can guess, bisect it, and test with your set square.

ANSWERS.

PART I.

EXERCISE II.

(1) 16 in. (2) 7 in. (3) 24 in. (4) 12 in. (5) 20 in. (6) 16 in.
(7) 34 in. (8) 8 in., 12 in., 20 in. (9) 16 min., 4 min., 8 min., 12 min.
(10) The latter by 4 in. (11) 28 in. (12) 5 in., 12 in., 14 in., 2 in.

EXERCISE III.

(1) 24 in., 18 in. (2) 20 in. (3) 16 in., 4 in. (4) 8 in. (5) 4, 4.
(6) 14 in., 7 min. (7) 5 in., 5 in., 10 in., 5 in.
(8) 10 in., 8 in., 10 in., 10 in., 12 in., 16 in.

EXERCISE IV.

(1) 2 in., 1 in., 6 in., 8 in. (2) 12 in., 4 in., 6 in., 2 in. (3) 3, 5, 6.
(4) 9 in., 3 mi. (5) $6\frac{1}{2}$ in. (6) 2 in., 3 in., 1 in. (7) 6 in., 8 in., 4 in.
(8) 4 ft.

EXERCISE V.

(1) 9s., 3s. (3) 4. (4) 9, 24. (5) 3 mi. (6) 2 in., 8 in., 6 in. (7) 4 in.
(8) Jack, by 4 mi. (9) Twice. (10) 20 min., 60 min.
(12) 14 in., 10 in., 4 in. (15) No. (19) $\frac{1}{4}$, $\frac{1}{3}$. (20) 2, twice.
(21) 4 in., 4 in.

EXERCISE VI.

(1) 18 in., 12 in., 6 in. (2) 4. (3) 18 in. (4) 6 in., 12 in. (5) 6 in., 6 in.
(6) Twice, 3 times. (7) 6 times, 3 times. (8) 9 in., 9 in.

EXERCISE VII.

(1) 1 in., $\frac{1}{4}$. (2) 2 or 3, 2. (3) $\frac{1}{4}$, $\frac{3}{4}$. (4) $1\frac{1}{2}$ in.
(5) $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{4}$; $2\frac{1}{2}$ in., $1\frac{1}{4}$ in.; 28 in., 23 in., $20\frac{1}{2}$ in.; 5 in., $2\frac{1}{2}$ in.
(6) Left half has been folded backward level with the right half; right half folded over across the left half; cross piece tucked under the downward piece. In order to have the two arms of nearly the same length.

EXERCISE VIII.

(1) $\frac{1}{4}$. (2) £$6\frac{1}{4}$. (3) $10\frac{1}{2}$d. (4) $\frac{3}{4}$ mile.
(5) 5, 9, 32, 35 halves; 10, 18, 64, 70 quarters. (6) 13, 23, 30, 73.
(7) $2\frac{1}{2}$, $1\frac{3}{4}$, $5\frac{1}{2}$, $4\frac{3}{4}$, $21\frac{1}{2}$, $15\frac{1}{4}$.

PART II.

EXERCISE I.

(1) 24. (3) 24, 32, 48; $13\frac{3}{4}$ in. (6) 11 in. (8) $\frac{1}{8}$, $\frac{1}{16}$, $\frac{3}{8}$, 2 in., 16 in.
(9) $\frac{1}{4}$; $\frac{1}{8}$, $\frac{1}{4}$; $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$. (11) All; $\frac{3}{4}$. (12) $\frac{3}{16}$; $\frac{1}{4}$, $\frac{1}{4}$; $\frac{1}{2}$, $\frac{1}{2}$.

EXERCISE II.

(5) $\frac{3}{4}$, $\frac{5}{8}$.

EXERCISE III.

(1) 15. (2) 3 in., 6 in. (3) 12 sq. in., 10 sq. in., 2 sq. in.
(4) 4 in. × 2 in.; 8 in. × 1 in. (5) 4 in. (6) 5 in., 20 in.
(7) 12 sq. in., 18 sq. in. (8) 15; 16 in. (9) 4, 2, 6.
(10) 16 sq. in., 16 sq. in., 4 sq. in., 20 sq. in. (12) 12 sq. in., 45 sq. in.
(13) 4 in., 2 in., 1 in. (14) 16 in., 24 in., 36 in.; 4. (15) $\frac{1}{4}$, 4.
(16) 6, 12 sq. in. (17) 480 sq. in.; 2d.; 120 sq. in., 360 sq. in.

EXERCISE IV.

(2) 4, 6. (3) 3s. (4) $2\frac{1}{2}$ sq. in., 10 sq. in., 2 sq. in. (5) Twice, half, half.
(6) 12, 48 sq. in. (7) $\frac{1}{8}$, $\frac{7}{8}$, 8 sq. in. (8) $10\frac{3}{4}$ in., $15\frac{1}{2}$ in.; 4 sq. in.
(9) $14\frac{9}{16}$ in.; 8 times. (10) $2\frac{1}{4}$, [illegible], $4\frac{1}{2}$, $4\frac{1}{2}$, 9, 27 sq. in. (11) 11 sq. in.
(12) $37\frac{1}{2}$, $7\frac{1}{2}$, 30 sq. in. (14) 312 sq. in. (15) 300 sq. in. (16) 30 sq. ft.
(17) $24\frac{1}{2}$ sq. ft. (18) 336, 216, 1,104 sq. in. (19) 117, 2,808 sq. in.
(20) 1,728 sq. in. (21) 4s. (22) 462 sq. in.

EXERCISE V.

(8) $2\frac{1}{2}$ in.; 6 in. (9) 3 sq. in., 7 in. (12) No; the oblong.

EXERCISE VI.

(1) 12, 6, 3, 24, 36, 60, 72. (2) 24, 72, 96, 12, 6. (3) 3, 5, 4, 2; 2 ft. 6 in.
(4) 2 ft. (5) 9. (6) 1 ft., $2\frac{1}{2}$ ft., 5 ft. (7) £1 16s.
(11) 2 ft. 3 in.; 2 ft.; 1 ft. 3 in.

PART III.

EXERCISE I.

(4) 15 whole and 5 halves; $17\frac{1}{2}$ sq. in. (5) 9·4 in., 14·1 in., 18·8 in.
(6) 12, 9, 21 ft. (8) 61·0. (9) 15 ft. (10) 19·5, 14·1, 38·4 ft.
(11) 570·5 yds. (12) 20·2s. (13) 19·8, 11·8 in. (14) 19·2 in.
(15) 6 whole sq. in.; 7·2 sq. in. (16) 3·3 in., 7·9 in. (19) 2s.

EXERCISE II.

(1) 1·4, 16 in. (2) 1·2 ft., 1·8 ft. (3) ·8 in. (4) 5·4, 1·6 in. (6) 4·9 in.
(7) 2·7 ft. (8) 16·7 yds. (9) 11·2d., 3·8d. (10) 2·5 in., 4·1 in.
(11) 4 in., 1·4 in. (12) 4 in., 2·8 in., 13·6 in., 1·2 in.

EXERCISE III.

(1) 32·2 sq. in., 11·6 in. (2) 41·2 ft., 103·2 sq. ft.
(3) 13·8, 25·2 sq. in.; 1·6, 11·2 sq. in. (4) 26 stamps.
(5) 29·4 sq. in.; 3 × 4·9 in., 15·8 in. (6) 5·4 sq. in., 9·6 in.
(7) 4·0, 8·6 sq. in.; 1·7, 3·4, 6·8 sq. in.; 4 in. (8) 16·8 sq. ft. (9) 90 sq. ft.
(10) 100 sq. ft., £4. (11) 13·5d., 10·7d. (12) 91·8 in.

EXERCISE IV

(1) 1·6 yds. (2) 1·6 in. (3) 4·5s. (4) 8. (5) £26·1. (6) 30. (7) 11 ft.
(8) 16·2, 6·4 sq. in. (9) 1,209 ·6 sq. in. (10) 4·6 in., 740 in., 552 in.
(12) 19·6d.

EXERCISE V.

(1) 1 in. (2) 2·83 sq. in. (6) 7·2, 8·9, 10·6 in. (7) 3·45, 9·45 in.
(8) 5·6, 6·6 in. (9) 3, $4\frac{1}{4}$, $1\frac{1}{4}$. (10) 52·4 mi. (11) 9, 14 yds.
(12) 14 mi., 8 mi.; $7\frac{3}{4}$.

EXERCISE VI.

(1) 11·2 in. (3) 1·8 sq. in. (6) 10 times, 2s. (12) 105 lb.

PART IV.

EXERCISE I.

(1) 240 sq. yds. (2) 9,600 sq. yds (3) 320 sq. yds. (4) 33 sq. yds. 2 sq. ft.
(5) 720 sq. in.; 4 sq. yds. 4 sq. ft. (6) 2,880 sq. in.; 2 sq. yds. 2 sq. ft.
(7) 100 yds., 164, 83 or 85, 66 ft.
(8) Hall = 150 sq. yds. 7 sq. ft.; A = D = 47 sq. yds. $2\frac{1}{4}$ sq. ft.; B = C = 49 sq. yds. 3 sq. ft.; each porch = 29 sq. yds. 5 sq. ft.; total floor space = 403 sq. yds. 1 sq. ft.; total area of site = 455 sq. yds. 5 sq. ft.
(9) 6 ft. 4 in., 9 ft., 2 sq. ft. 80 sq. in.; 64 sq. in. (10) 792 sq. in.
(11) 28,188 sq. in. (12) 10 sq. yds. 29 sq. in.
(13) 721 sq. yds. 2 sq. ft. 59 sq. in. (14) 208. (15) 432 bricks.
(16) 30 lb., 800 sq. ft.

EXERCISE II.

(1) 45 yds. high. (2) 440. (3) 258 yds. (4) 14 ft. 8 in.
(5) 1,984 sq. yds. (6) 14 in.; 1 sq. ft. 52 sq. in. (7) 2 ft. (8) 48 ft.
(9) $907\frac{1}{2}$ sq. yds. (10) 3,520 times. (11) 121 plots. (12) 12.

EXERCISE III.

(4) $\frac{1}{6}$. (5) 19s. 10d.; 238d. (6) 2s. 8d.
(7) 6 ft. 3 in. long; 3 ft. $4\frac{1}{2}$ in. wide. (8) 15. (9) 39 sq. in.; 10 sq. in.
(10) $\frac{1}{3}$; $\frac{3}{14}$; $1\frac{2}{3}$. (11) $37\frac{1}{2}$, 1, 4, 8 sq. in. (12) 12, 6, 10 in.

EXERCISE IV.

(4) 17·49. (5) 116·64 mi. (6) 17·43 in. (7) 2·5s. (8) 17·55 mi.
(9) 9·75 tons. (10) 2 cwt. 2 qrs. 17 lbs. (11) 40461·68. (12) $1\frac{9}{20}$.
(13) 6·17 in. (14) 42. (15) £21·12.

Printed by Sir Isaac Pitman & Sons, Ltd., Bath.
N—(747)